LECTURE OUTLINE
Taylor Approximation

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Math 15

Oct. 6, 2004

Explore Pendulum

Taylor Approximation

Utilizing Energy Conservation: The Pendulum

Suppose we have an ideal pendulum of length L (in a vacuum) and from rest intend to impart it with a velocity of $v_0 \frac{m}{sec}$.

1. Find the potential energy of our pendulum for each angle θ.

2. Use conservation of energy to find the the pendulum's angular speed at each angle θ.

3. How fast must we start our pendulum so that it makes a complete circle?

4. For each v_0, what is our pendulum's maximum height?

Tangent Line Approximation

Near a

$$f(x) \approx f(a) + f^1(a)(x-a) \equiv P_1(x,a).$$

Example: Approximate $\sqrt{1.01}$.

Quadratic Approximation

Even better, near a

$$f(x) \approx f(a)+f^1(a)(x-a)+\frac{1}{2}f^2(a)(x-a)^2 \equiv P_2(x,a).$$

Example: Better approximate $\sqrt{1.01}$.

nth Order Approximation at a

$$f(x) \approx \sum_{k=0}^{n} \frac{f^k(a)}{k!}(x-a)^k \equiv P_n(x,a)$$

near a. Notice $\frac{d^k}{dx^k} P_n(x,a)\Big|_{x=a} = f^k(a)$ for all $0 \le k \le n$.

Ex: Find $P_n(x,0)$ for $\sin(x)$.