LECTURE OUTLINE

Total Derivative and Chain Rule

Professor Leibon

Math 15

Oct. 29, 2004

Tangent Planes

The Chain Rule

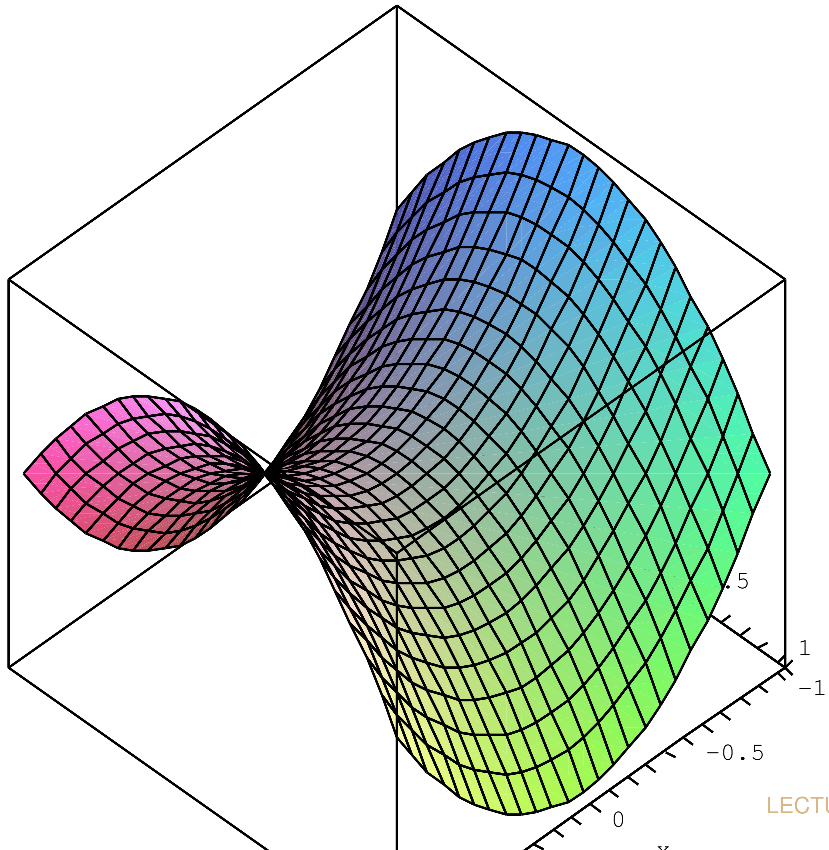
Linear Approximation

Continuity

Tangent Plane (from last time)

The tangent plane at a point is given by $\vec{n} \cdot (\vec{r} - \vec{p})$ with $\vec{n} = -\frac{\partial f}{\partial x}\hat{i} - \frac{\partial f}{\partial y}\hat{j} + \hat{k}$ and $\vec{p} = x_0\hat{i} + y_0\hat{j} + f(x_0, y_0)\hat{k}$.

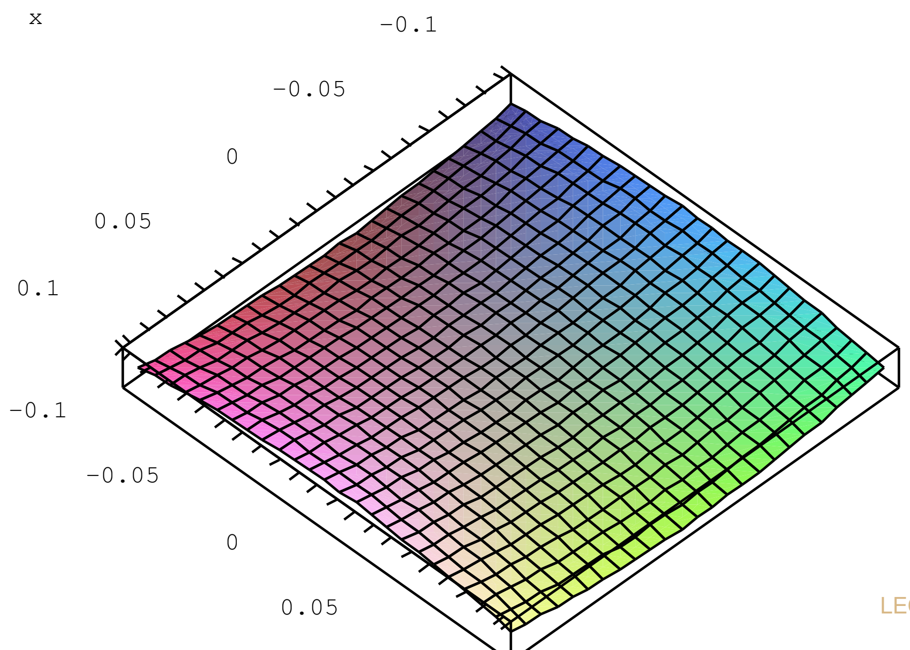
Example: $f(x, y) = x^2 - y^2$ at $(0, 0, 0)$.



Tangent Plane

The tangent plane at a point is given by $\vec{n} \cdot (\vec{r} - \vec{p})$ with $\vec{n} = -\nabla f + \hat{k}$ and $\vec{p} = x_0\hat{i} + y_0\hat{j} + f(x_0, y_0)\hat{k}$.

Example: $f(x, y) = x^2 - y^2$ at $(0, 0, 0)$. Zoom in towards $(0, 0, 0)$....



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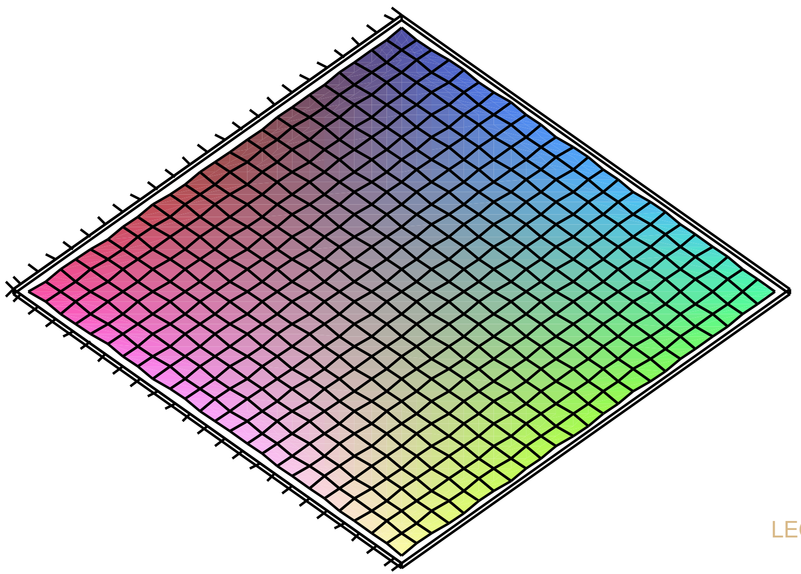
Example: $f(x, y) = x^2 - y^2$ at $(0, 0, 0)$. Zoom in towards $(0, 0, 0)$ and we see this plane.

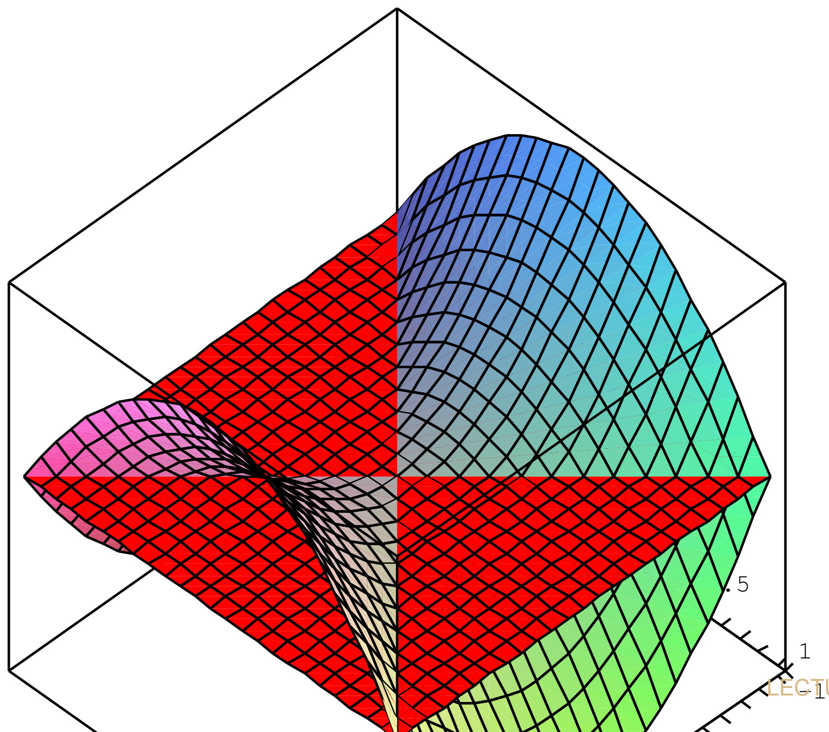


Tangent Plane

In other words: near (x_0, y_0) we have that $f(x, y)$ looks like

$$z = f(x_0, y_0) + \nabla f \cdot (x - x_0, y - y_0).$$

Example: $f(x, y) = x^2 - y^2$ near $(0, 0, 0)$.



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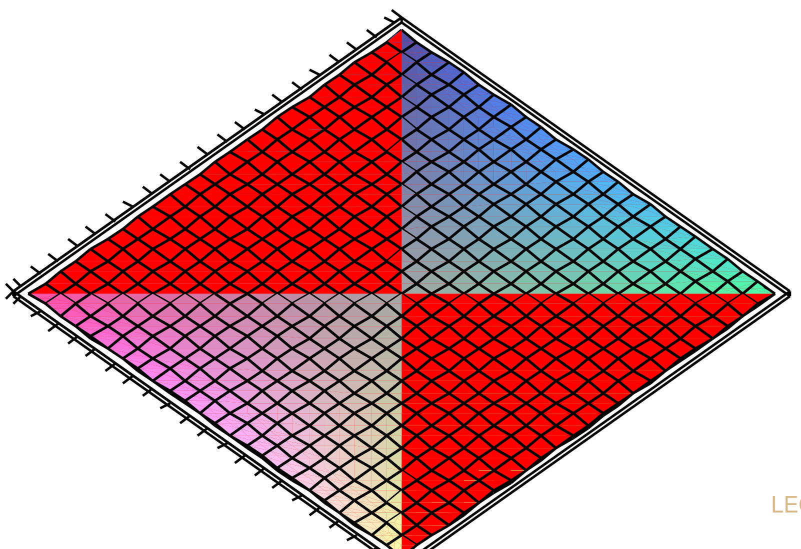


A Cruel and UNUSUAL view of the Chain Rule

Recall from last time

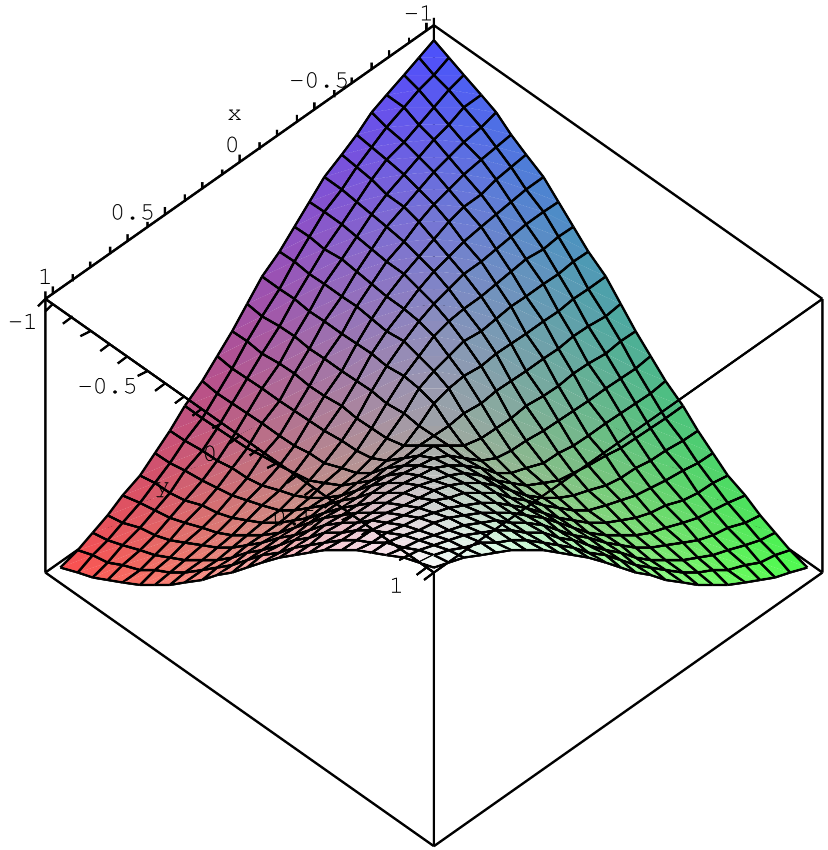
$$\frac{df(x,y)}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}.$$

This requires a caveat. The usual caveat is that this is true provided $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are continuous. In other words, be careful when a denomenator takes on a zero, or when function can't make up its mind about a certain value.

Ex. Let $f(x,y) = \frac{xy}{\sqrt{x^2+y^2}}$, $x(t) = \cos(\theta_0)t$ and $y(t) = \sin(\theta_0)t$. Compare $\frac{df}{dt}$ and $\frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$.

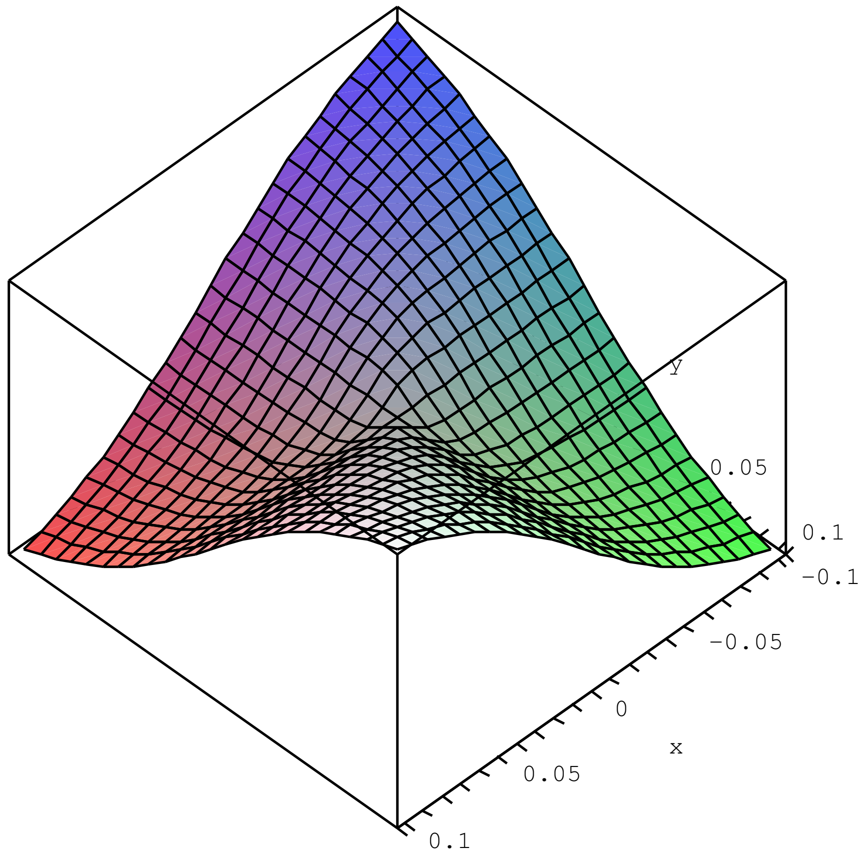
The Non-Tangent Plane

Let $f(x,y) = \frac{xy}{\sqrt{x^2+y^2}}$. Zoom in towards the $(0,0,0)$...



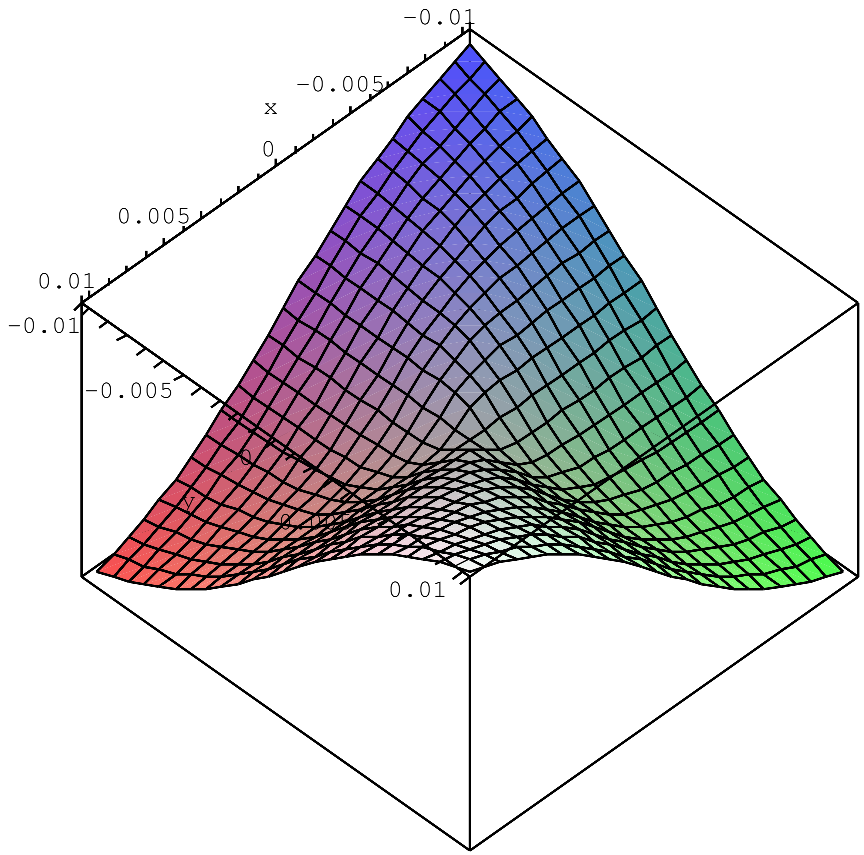
The Non-Tangent Plane

Let $f(x,y) = \frac{xy}{\sqrt{x^2+y^2}}$. Zoom in towards the $(0,0,0)$, and,...



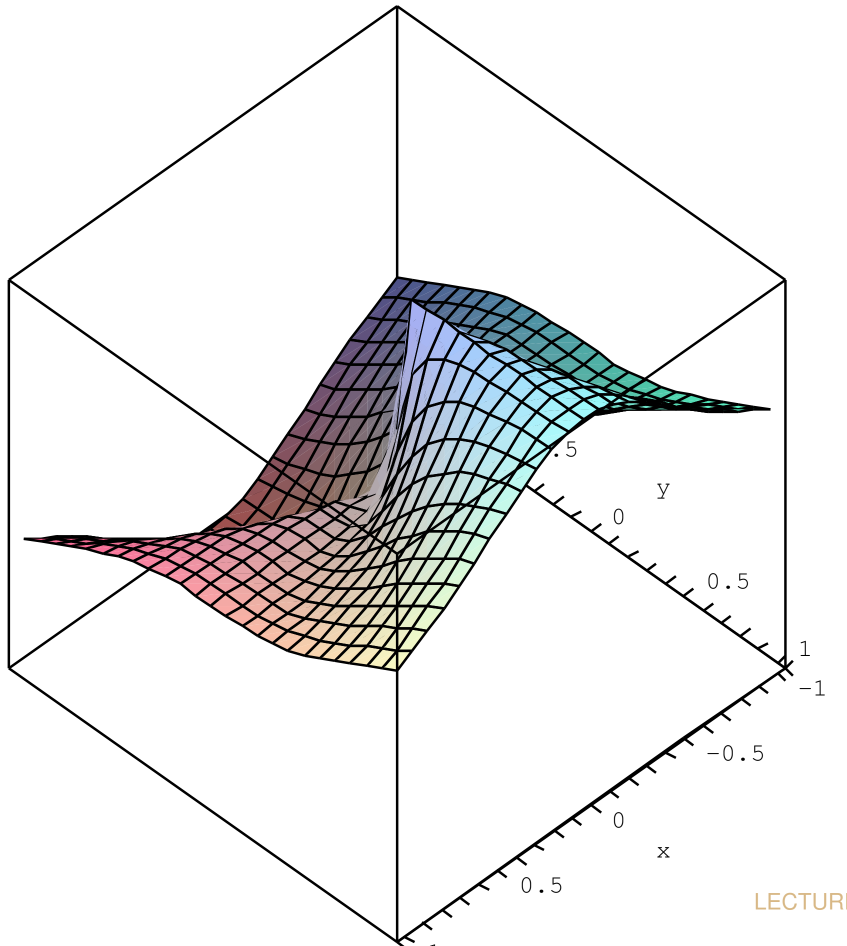
The Non-Tangent Plane

Let $f(x, y) = \frac{xy}{\sqrt{x^2+y^2}}$. Zoom in towards the $(0, 0, 0)$, and nothing happens!



The Non-Tangent Plane

Let $f(x, y) = \frac{xy}{\sqrt{x^2+y^2}}$ and zoom in towards $(0, 0, 0)$ on the graph of $\frac{\partial f}{\partial x}$...



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The Non-Tangent Plane

Let $f(x, y) = \frac{xy}{\sqrt{x^2+y^2}}$ and zoom in towards $(0, 0, 0)$ on the graph of $\frac{\partial f}{\partial x}$ and EEEEEKKKK!!!!!