EX 155 The answers here are gotten by inspection.

1.) @ $(0,.2)$, $(0,2)$, & $(-1.8,-2)$

2.) near $(.2, 2.2)$

3.) i) from $(-1,1)$, left $((-1,0))$.
ii) around $(.4, 2.2)$.

4.) Yes, around $(0,1)$ and $(-1.4, 2)$.

5.) $-\frac{\pi}{4}$ $\frac{3\pi}{4}$

$\frac{5\pi}{4}$ $\frac{\pi}{4}$

π 0

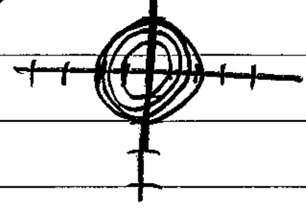
EX 156 A: $4 = 4 - x^2 - y^2 \Leftrightarrow x^2 + y^2 = 0 \Leftrightarrow x^2 = -y^2 \Leftrightarrow x = y = 0$

$3 = 4 - x^2 - y^2 \Leftrightarrow x^2 + y^2 = 1$

$2 = 4 - x^2 - y^2 \Leftrightarrow x^2 + y^2 = 2$

$1 = 4 - x^2 - y^2 \Leftrightarrow x^2 + y^2 = 3$

$0 = 4 - x^2 - y^2 \Leftrightarrow x^2 + y^2 = 4$



The level curves are all circles centered @ the origin.

EX 157

1. $\frac{\partial g}{\partial x} = 2x, \quad \frac{\partial g}{\partial y} = 2y$

2. $\frac{\partial h}{\partial x} = \cos(x) \quad \frac{\partial h}{\partial y} = -\sec^2 y$

3. $\frac{\partial g}{\partial x} = \frac{\cos(x)}{\sqrt{x^3+y}} + \sin(x)\cdot\left(\frac{-1}{2}\right)\cdot(x^3+y)^{-\frac{3}{2}}\cdot 3x^2$

(can simplify if you so desire)

$\frac{\partial g}{\partial y} = \sin(x)\cdot\left[\left(\frac{-1}{2}\right)\cdot(x^3+y)^{-\frac{3}{2}}\right]$
(ditto)

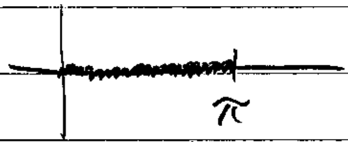
4. $\frac{\partial q}{\partial r} = 2r\cdot(\cos(3\theta) + \sin^2(5\theta))$
(ditto)

$\frac{\partial q}{\partial \theta} = r^2(-3\sin(3\theta) + 2\cdot\sin(5\theta)\cdot\cos(5\theta)\cdot 5)$
(ditto)

5. $\frac{\partial q}{\partial x} = y + 2z^5xy^3 - 4zx^3, \quad \frac{\partial q}{\partial y} = x + 3z^5x^2y^2, \quad \frac{\partial q}{\partial z} = 5z^4x^2y^3 - x^4$

6. $\frac{\partial x}{\partial r} = \cos(\theta) \quad , \quad \frac{\partial x}{\partial \theta} = -r\sin\theta$

$\boxed{\text{EX 158}}$



f is a dumb symbol for temperature — so let's use T.

temp: $T(x,t) = e^{-t}\sin(x)$

i) We will need to have explicit formulas for the partial derivatives of T.

$$\frac{\partial T}{\partial x} = e^{-t}\cos(x) \quad \Big\} \quad \frac{\partial T}{\partial t} = -e^{-t}\cos(x),$$

The book wants:

a) i) $\frac{\partial T}{\partial t}(0,1) = -e^{-1}\cancel{\cos(0)} = \frac{-1}{e}$

$\frac{\partial T}{\partial t}(\frac{\pi}{3},1) = -e^{-1}\cdot\frac{1}{2} = \frac{-1}{2e}$

$\frac{\partial T}{\partial t}(\frac{\pi}{2},1) = 0.$

b) $\frac{\partial T}{\partial x}(\frac{\pi}{3},1) = e^{-1}\cdot\frac{1}{2}$, $\frac{\partial T}{\partial x}(\frac{\pi}{2},1) = 0$, $\frac{\partial T}{\partial x}(\frac{-\pi}{3},1) = \frac{-1}{2e}$

c) $\frac{\partial T}{\partial x}(\frac{\pi}{4},1) = \frac{-1}{\sqrt{2}\,e}$, $\frac{\partial T}{\partial x}(\frac{\pi}{3},1) = \frac{-1}{2e}$, $\frac{\partial T}{\partial x}(\frac{\pi}{2},1) = 0$

ANS: @ $\pi/3$.

d) Yes. (To me, @ least.)

EX 159 | $SA = 2\pi r h$.

NB: In mathematics, "cylinder" excludes the endcaps; thus do I proceed.

$SA_i = 2\pi r h$ | $SA_f = 2\pi(r+\Delta r)(h+\Delta h)$

$= 2\pi r h + 2\pi r \Delta h + 2\pi h \Delta r + 2\pi \Delta r \Delta h$

$\Delta SA := SA_f - SA_i = 2\pi(r+\Delta r)(h+\Delta h) - 2\pi r h$

$= 2\pi \Delta r \Delta h + 2\pi h \Delta r + 2\pi r \Delta h$

← It changes that much, exactly.

Now, for the physics book way:

$\dfrac{\partial SA}{\partial h} = 2\pi r$ | $\dfrac{\partial SA}{\partial r} = 2\pi h$

← varying h only

$\Rightarrow \Delta SA \approx \dfrac{\partial SA}{\partial h} \cdot \Delta h = 2\pi r \Delta h$

← varying r only

& $\Delta SA \approx \dfrac{\partial SA}{\partial r} \cdot \Delta r = 2\pi h \Delta r$

$\Rightarrow \Delta SA \approx 2\pi r \Delta h + 2\pi h \Delta r$.

The moral of the story is that physicists like to pretend that $\Delta r \Delta h = 0$.

EX $\boxed{60}$ The book is asking for

$$\frac{\partial f}{\partial t}\left(\frac{\pi}{12}, \frac{\pi}{6}\right) = -5\sin(3x)\cos(2t)\cdot 2\Big|_{\left(\frac{\pi}{12}, \frac{\pi}{6}\right)}$$

$$= -10\cdot\sin\left(\frac{\pi}{4}\right)\cdot\cos\left(\frac{\pi}{3}\right) = -10\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{2} = \frac{-5}{\sqrt{2}}$$

The slope of the string @ that point would be...

$$\frac{\partial f}{\partial x}\left(\frac{\pi}{2}, \frac{\pi}{6}\right) = -15\sin(2t)\cos(3x)\Big|_{\left(\frac{\pi}{2}, \frac{\pi}{6}\right)} = -15\,\frac{\sqrt{3}}{2}\cdot 0 = 0.$$

EX 61 They're asking for $\frac{\partial T}{\partial t}(0)$. Need $x(t), y(t), \& z(t)$.

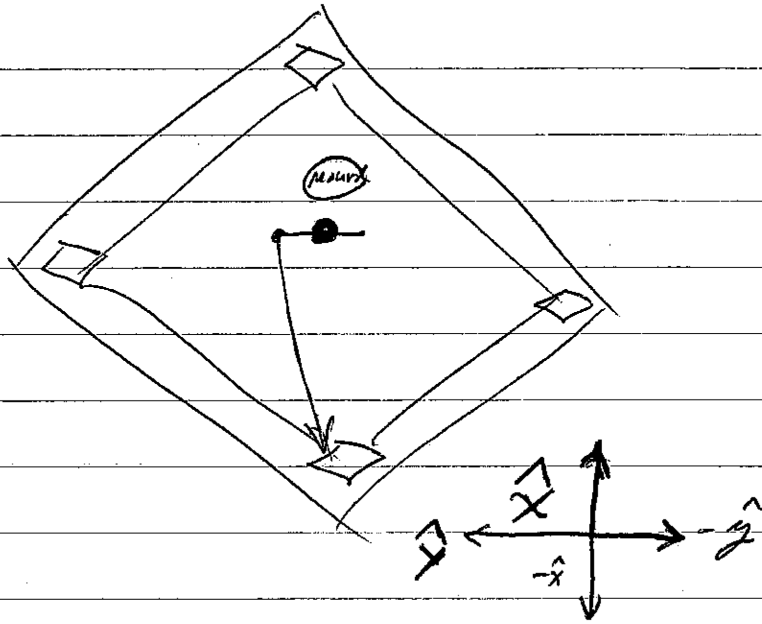
$x(t)=2$, $y(t)=1$, & $z(t)=3+5t$.

Thus $T(t)=e^{-4-1-9-30t-25t^2} = e^{-25t^2-30t-14}$

$\Rightarrow \frac{\partial T}{\partial t}(0) = (-50t-30)\,e^{-25t^2-30t-14}\Big|_{(0)} = (-30)\cdot e^{-14}$.

P3A (a) Draw a picture:

Right-handed.



(b) $\vec{a}(t) = -32\hat{z}$

$\Rightarrow \vec{V}_c(t) = -32t\hat{z} + \vec{K}_1$ But $\vec{V}_c(0) = -120\hat{x} - \frac{4}{3}\hat{y} + 8\hat{z}. = \hat{K}_1$

$\Rightarrow \vec{c}(t) = -16t^2\hat{z} - 120t\hat{x} - \frac{4}{3}t\hat{y} + 8t\hat{z} + \hat{K}_2$, but

$\vec{c}(0) = 90\hat{x} + \hat{y} + 5\hat{z} = \hat{K}_2$,

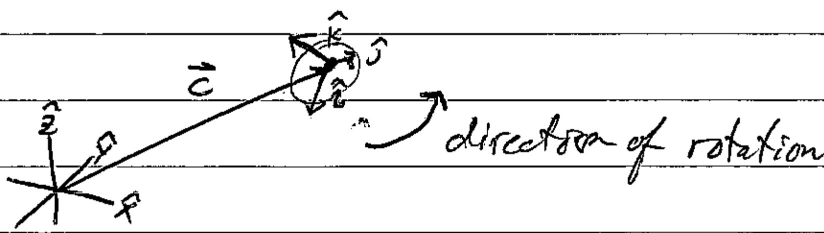
So $\vec{c}(t) = (-16t^2 + 8t)\hat{z} + -120t\hat{x} + -\frac{4}{3}t\hat{y} + 90\hat{x} + \hat{y} + 5\hat{z}$.

(c) Well, let's solve for when $x = 0$.

$\Rightarrow -120t\hat{x} + 90\hat{x} = 0 \Rightarrow t = \frac{3}{4} \Rightarrow y = -\frac{4}{3}\cdot\frac{3}{4} + 1 = 0$
& $z = -16\cdot\frac{9}{16} + 8\cdot\frac{3}{4} + 5 = -9 + 6 + 5 = 2$

Sounds like the strike zone to me.

P3 B] Picture!



(d) $r = .25$, $\theta(t) = 6 \cdot 2\pi t = 12\pi t$

So $\vec{r}_c(t) = r\hat{r}$. (like always...?)

(e) $\vec{r}_c(t) = .25\cos(12\pi t)\hat{\imath} + .25\sin(12\pi t)\hat{\jmath}$

(f) $\vec{r}(t) = \vec{c}(t) + \vec{r}_c(t) =$

$$(-16t^2 + 8t + 5)\hat{z} + (-\tfrac{4}{3}t + 1)\hat{y} + (-120t + 90)\hat{x} + \vec{r}_c(t)$$

$$= \sim\!\sim + \tfrac{1}{4}\cos(12\pi t)\cdot\tfrac{1}{\sqrt{2}}(-\hat{x} - \hat{z}) + \tfrac{1}{4}\sin(12\pi t)\cdot\hat{k}\times\hat{\imath}$$

$$\hat{k}\times\hat{\imath} = \frac{1}{\sqrt{3}}\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ -1 & 1 & 1\\ -1 & 0 & -1\end{vmatrix}\cdot\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{6}}(\hat{x} - 2\hat{y} - \hat{z})$$

$$= \left(-16t^2 + 8t + 5 - \frac{1}{4\sqrt{2}}\cdot\cos(12\pi t) - \frac{1}{4\sqrt{6}}\sin(12\pi t)\right)\hat{z}$$

$$+ \left(-\tfrac{4}{3}t + 1 - \sqrt{\tfrac{2}{3}}\,\tfrac{1}{\cancel{4}}\sin(12\pi t)\right)\hat{y}$$

$$+ \left(-120t + 90 - \frac{1}{4\sqrt{2}}\cos(12\pi t) + \frac{1}{4\sqrt{6}}\sin(12\pi t)\right)\hat{x}$$

P3B (g) Well, the ball would slowly stop spinning, it would fall slightly in front of the home plate, and its path would not be parabolic.

All of that is due to air resistance.