

LECTURE OUTLINE

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Rotational Derivatives

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Math 15

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Last Time: Using the Diana Convention

For our rotation use \hat{i} , \hat{j} , and \hat{k} with \hat{k} the axis of rotation and \hat{i} to \hat{j} in the direction of rotation (right hand rule). Give \hat{r} and $\hat{\theta}$ their usual meanings with respect to \hat{i} , \hat{j} , \hat{k} . There is a second "view" we might take to think about the center (center of mass) which we might call \hat{x} , \hat{y} , \hat{z} . A particle moving about another particle can be described by $\vec{r}_T = \vec{c} + r\hat{r} + z\hat{k}$.

Ex: I have a foot long football with radius $\frac{1}{4}$ ft around its central axis. Under ideal conditions, I punt my football with initial position $3\hat{z}$ feet, initial velocity $5\hat{x} + 3\hat{z}$ feet per second, and impart it a clockwise rotation of 4 revolution per second about the \hat{y} -axis (as was pointed out, I am not a good punter). Find equation of motion for the tip of the ball in the above notation.

$$\begin{aligned}\vec{r} &= \left(5t\hat{x} + \left(-\frac{gt^2}{2} + 3t + 3 \right) \hat{z} \right) + \frac{1}{2} \vec{r}(8\pi t) \\ &= \left(5t - \frac{\cos(8\pi)}{2} \right) \hat{x} + \left(-\frac{gt^2}{2} + 3t + 3 + \frac{\sin(8\pi t)}{2} \right) \hat{z}\end{aligned}$$

Books Usual Assumptions

Usual Book Assumptions:

(1) $\dot{r} = 0$ (Not Nutty)

(2) $z = 0$ (Not so Nutty)

(3) $\frac{d\hat{k}}{dt} = 0$ (Nutty)

Note: (3) The Great Yo-Yo Restriction allows us to assume $\frac{d\hat{i}}{dt} = 0$ and $\frac{d\hat{j}}{dt} = 0$ as well.

Deriving the Formula

Book's Notation under the Usual Book Assumptions:

$$\vec{\omega} \equiv \dot{\theta} \hat{k} \equiv \omega \hat{k}$$

$$\vec{\alpha} \equiv \frac{d\vec{\omega}}{dt} = \ddot{\theta} \hat{k}$$

$$\vec{v} \equiv \frac{d\vec{r}}{dt} = r\dot{\theta} \hat{\theta} = \vec{\omega} \times \vec{r}$$

$$\vec{a} \equiv \frac{d\vec{v}}{dt} = -r\dot{\theta}^2 \hat{r} + r\ddot{\theta} \hat{\theta} = \vec{\omega} \times \vec{v} + \vec{\alpha} \times \vec{r}$$

Derivatives in Polar Coordinates

$$\frac{d\hat{r}}{dt} = \dot{\theta}\hat{\theta}$$

$$\frac{d\hat{\theta}}{dt} = -\dot{\theta}\hat{r}$$

$$\frac{d\vec{r}}{dt} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$\frac{d^2\vec{r}}{dt^2} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

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Is $8\pi\hat{y}$ equal to $\vec{\omega}$?

Use the cross product to find the velocity and acceleration of the point on the tip of our football relative to the center of mass.

Find the velocity and acceleration of the point on the tip of our football.