LECTURE OUTLINE
Rotational Kinematics Examples

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Math 15

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Goals

Cross Product Application

Rigid Body Kinematics

Application to spacial reasoning

Ex: Note we can think of a line as $t\vec{v} + \vec{p}$. Define what it means for two lines to be non-parallel, and find a formula for the distance between two non-parallel lines.

Find the distance between the lines $(\hat{i} - 2\hat{k})t + \hat{i}$ and $\vec{v} = t(\hat{i} + \hat{j} + \hat{k})$.

Last Time

For our rotation use $\hat{i}$, $\hat{j}$, and $\hat{k}$ with $\hat{k}$ the axis of rotation and $\hat{i}$ to $\hat{j}$ in the direction of rotation (right hand rule). Give $\hat{r}$ and $\hat{\theta}$ their usual meanings with respect to $\hat{i}$, $\hat{j}$, $\hat{k}$. There is a second "view" we might take to think about the center (center of mass) which we might call $\hat{x}$, $\hat{y}$, $\hat{z}$. A particle moving about another particle can be described by $\vec{r}_T = \vec{c} + r\hat{r} + z\hat{k}$.

Ex: I have a foot long football with radius $\frac{1}{4}\ ft$ around its central axis. Under ideal conditions, I punt my football with initial position $3\hat{z}$ feet, initial velocity $5\hat{x} + 3\hat{z}$ feet per second, and impart it a clockwise rotation of 4 revolution per second about the $\hat{y}$-axis (as was pointed out I am not a good punter). Find equation of motion for the tip of the ball in the above notation.

Kinematics

Usual Book Assumptions:

(1) $\dot{r} = 0$ (Not Nutty)

(2) $z = 0$ (Not so Nutty)

(3) $\frac{d\hat{k}}{dt} = 0$ (Nutty)

Note: (3) **The Great Yo-Yo Restriction** allows us to assume $\frac{d\hat{i}}{dt} = 0$ and $\frac{d\hat{j}}{dt} = 0$ as well.

Kinematics

Book's Notation under the Usual Book Assumptions:

$$\vec{\omega} \equiv \dot{\theta}\hat{k} \equiv \omega\hat{k}$$

$$\vec{\alpha} \equiv \frac{d\vec{\omega}}{dt} = \ddot{\theta}\hat{k}$$

$$\vec{v} \equiv \frac{d\vec{r}}{dt} = r\dot{\theta}\hat{\theta} = \vec{\omega} \times \vec{r}$$

$$\vec{a} \equiv \frac{d\vec{v}}{dt} = -r\dot{\theta}^2\hat{r} + r\ddot{\theta}\hat{\theta} = \vec{\omega} \times \vec{v} + \vec{\alpha} \times \vec{r}$$

Derivatives in Polar Coordinates

$$\frac{d\hat{r}}{dt} = \dot{\theta}\hat{\theta}$$

$$\frac{d\hat{\theta}}{dt} = -\dot{\theta}\hat{r}$$

$$\frac{d\vec{r}}{dt} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$\frac{d^2\vec{r}}{dt^2} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$