LECTURE OUTLINE
Practice Exam

Professor Leibon

Math 15

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Recall

$$\frac{d\hat{r}}{dt} = \dot{\theta}\hat{\theta}$$

$$\frac{d\hat{\theta}}{dt} = -\dot{\theta}\hat{r}$$

$$\frac{d\vec{r}}{dt} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$\frac{d^2\vec{r}}{dt^2} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

$$\int \frac{1}{1+x^2}dx = \arctan(x) + C$$

Problem 1-4 (Random and Slightly Modified HW - no friction or polar)

(Ex. 54) The coordinates of a vector in three-dimensional space is (a, b, c).

(a) Express the vector as a sum of scalar multiples of the standard basis vectors.
(b) Find the magnitude (norm) of this vector.
(c) Find a unit vector in the direction of the given vector.

Problem 5 (Random and Slightly Modified Book Example)

(Ex. 20) An object is moving around the circle $x^2 + y^2 = 4$ in the x, y plane (where the unit of distance is measured in meters) in a clockwise direction at a constant speed of 2 meters per second. Assume its initial position is $2\hat{i}$.

(a) Find its position after 5 seconds.

(b) Find a vector representing its velocity when it is located at the point with position vector

$\sqrt{2}\hat{i} + \sqrt{2}\hat{j}$.

Problem 6 (Slightly Modified Class Example: Cycloid, Ellipse, Pendulum....)

Suppose we have an ellipse and know that

$$\frac{d}{dt}\vec{r}(t) = \frac{e^2 d \sin(t)}{(1 + e\cos(t))^2}\hat{r} + \frac{ed}{1 + e\cos(t)}\hat{\theta}$$

Express this vector in Cartesian Coordinates.

Problem 7 (Theory Based: dot product rules, conservation of energy,work def...)

Use the definition of dot product and the rules of one dimensional calculus to justify that

$$\frac{d}{dt}(\vec{v}\cdot\vec{w}) = (\frac{d}{dt}\vec{v})\cdot\vec{w} + \vec{v}\cdot(\frac{d}{dt}\vec{w}).$$

Problem 8: **Synthesis of material** *My Choice!*

The force of gravity exerted by a massive object of mass M located at the origin of our three-dimensional axes on a small object of mass m located a distance r from the origin has magnitude $\frac{mMg}{r^2}$ and acts directly towards the origin. Consider the work done by this force on the small object as it moves from the point $(0, 4, 0)$ to $(2, 0, 0)$ in a straight line.

(a) Without computation, should this work be positive or negative? Explain.
(b) Compute the work.
(c) Suppose I told you that this force was a conservative. Find a second, easier way to compute this force with this information. Do the computation.