LECTURE OUTLINE

Perturbation

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Math 15

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Goal

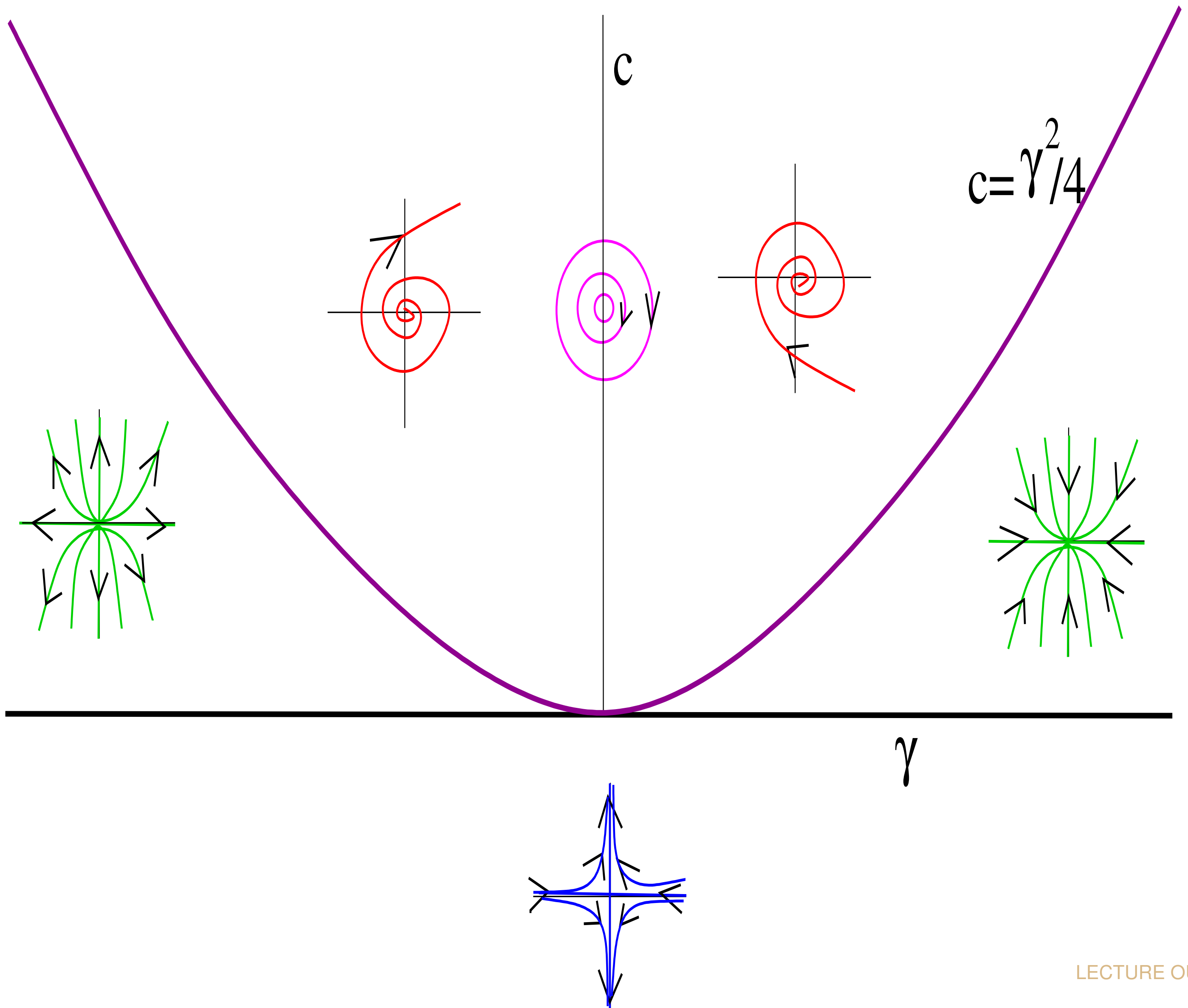
The Magenta Parabola

Approximation of Force at

Equilibrium

Resonance

Solutions to $\frac{d^2 f}{dt^2} + \gamma \frac{df}{dt} + cf = 0.$



The Magenta Parabola

Find a two solutions to $\frac{d^2 f}{dt^2} + \gamma \frac{df}{dt} + \frac{\gamma^2}{4} f = 0$. This is called the *Critically Damped* case.

The famous guessing method: If at first you don't succeed, then **multiply by** t^k and try try again.

Approximation

Many physical systems can be approximated by a harmonic oscillator. For example, suppose we have a one dimensional conservative force $F(x) = -\nabla U(x)$. A point a is an equilibrium point if it stays put, in other words $F(a) = 0$. We are often interested in what happen when we *perturb* a equilibrium solution. By Taylor approximation, near a we have we have

$$m\frac{d^2(x-a)}{dt} \approx F(a) + \frac{\partial F}{\partial x}(a)(x-a) \approx \frac{\partial F}{\partial x}(a)(x-a).$$

Frequency of Small Oscillations

Example (Leonard-Jones Model): What is the frequency of small oscillations about equilibrium (the van der Waals radius) of a "particle of mass m" in a potential

$$U(r) = \frac{a}{r^{12}} - \frac{b}{r^6}$$

for a and b positive constants.

Exercise: Molecular Bonding Approximation

Exercise 1: What is the frequency of small oscillations about equilibrium of a "particle of mass m" in a potential

$$U(r) = \frac{2}{r^2} + r^2.$$

Exercise: Molecular Bonding Approximation

Exercise 2: (The Morse curve approximation in the covalent case) Let

$$U(r) = A\left(e^{-2\alpha(r-r_0)} - 2e^{-\alpha(r-r_0)}\right)$$

were A,α, and r_0 are posiitve constants particular to the molecule. What is the frequency of small oscillations about equilibrium ?

Resonance

Find a general solution to

$$D^2 f + \gamma D f + c f = \alpha \sin(\omega t)$$

Suppose the system is initially at rest and the above driving force is applied. Describe the system's behavior. What does the system look like for large time?