POLAR COORDINATE PROBLEMS

Math 15

Oct. 20, 2004

Goal

Polar Coordinate Review

Polar Coordinate Problems

Polar Coordinates

We define polar coordinates via

$$(r, \theta)_P = (r\cos(\theta), r\sin(\theta)).$$

We can find a polar coordinate determining (x, y) via

$$(x, y) = \left(\sqrt{x^2 + y^2}, \arctan\left(\frac{y}{x}\right)\right)_P.$$

Polar Coordinates: Vectors

When thinking in terms of polar coordinates, we use $\hat{r}$ to describe position

$$\vec{r} = r\hat{r}(\theta) = r(\cos(\theta)\hat{i} + \sin(\theta)\hat{j}),$$

and use $\hat{r}$'s perpendicular companion

$$\hat{\theta} = -\sin(\theta)\hat{i} + \cos(\theta)\hat{j}$$

to describe vectors at $(r, \theta)_P$.

Derivatives in Polar Coordinates

$$\frac{d\hat{r}}{dt} = \dot{\theta}\hat{\theta}$$

$$\frac{d\hat{\theta}}{dt} = -\dot{\theta}\hat{r}$$

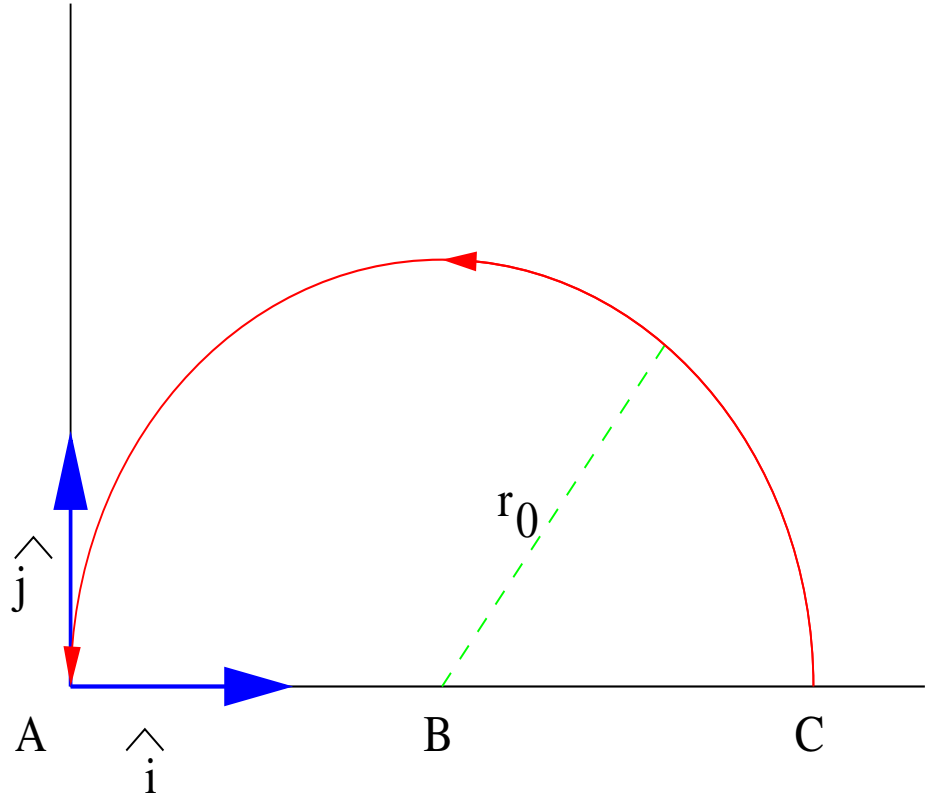
$$\frac{d\vec{r}}{dt} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$\frac{d^2\vec{r}}{dt^2} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

Problem 1

A particle travels on a circle of constant radius $5m$ from C to A around B. Suppose the component of its acceleration vector that points in the tangential direction has magnitude equal to t^3, and that our particle begins its journey with a velocity of 0 meters per second.

(a) Express the position vector in Cartesian and Polar coordinates using B as the origin.
(b) Is there an acceleration in the $\hat{r}$ direction? If so find it.



Problem 2

Imagine a particle travels around a circle of changing radius with a constant speed in the $\hat{\theta}$ direction of ω. Suppose it starts at $\hat{i}$ and that the radius of the circle is changing with time as $r(t) = 1 + t^2$. (Hint: $\frac{d \arctan(x)}{dx} = \frac{1}{1+x^2}$.)

(a) Express $\vec{r}(t)$ in Cartesian coordinates.

(b) Find $r(\theta)$, the distance from the origin as a function of angle.

(c) How large must ω be to insure that our particle will eventually make a full revolution?