P1] (a) First, we get the polar coordinates.

Getting θ as a function of t is the hard step, so we get it first.

By our given formula sheet,

$$\frac{d^2\vec{r}}{dt^2} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

We are told that $|2\dot{r}\dot{\theta} + r\ddot{\theta}| = t^3$.

But $r(t) = 5$, so $\dot{r}(t) = 0 \Rightarrow$

$|5\ddot{\theta}| = t^3$. It would be nice to know that $\ddot{\theta} \geq 0$, but this is already clear because $\ddot{\theta}$ must have the same sign as the component of the acceleration in the tangential direction, which is ≥ 0 since $t \geq 0$.

Thus $5\ddot{\theta} = t^3 \Rightarrow 5\dot{\theta} = \frac{1}{4}t^4 + C_1$

But $\vec{v}(0) = 0 \Rightarrow \dot{\theta}(0) = 0 \Rightarrow C_1 = 0$.

$\Rightarrow 5\theta = \frac{1}{20}t^5 + C_2 \Rightarrow 0 = 0 + C_2$

$\Rightarrow \theta = \frac{t^5}{100}$

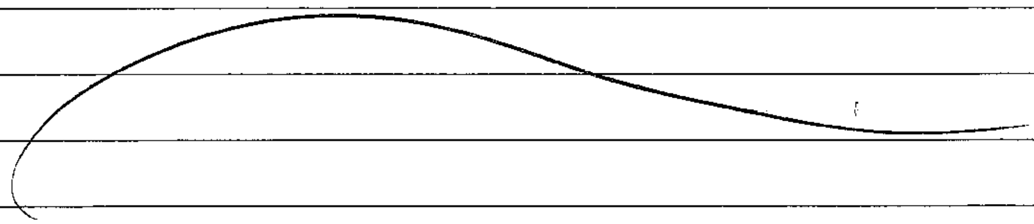
P1] (a) ctd $\Rightarrow$ polar coordinates are $(r,\theta)_p = \left(5, \frac{t^5}{100}\right)_p$

cartesian coordinates are $(x,y) = \left(5\cos\left(\frac{t^5}{100}\right), 5\sin\left(\frac{t^5}{100}\right)\right)_c$.

(b) acceleration in $\hat{r}$ direction: $\ddot{r} - r\dot{\theta}^2 = 0 - 5\cdot\left(\frac{t^4}{20}\right)^2$

$= -\frac{1}{80} t^8$.

... so, yes. That's it.



P2 (a) We are given r, so we need to find θ.

We are told that $|\text{proj}_{\hat{\theta}} \vec{v}| = \omega$, a constant.

(by formula sheet)

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

Thus $\omega = r\dot{\theta} \Rightarrow \dot{\theta} = \frac{\omega}{r} = \frac{\omega}{1+t^2}$

$$\Rightarrow \theta = \int_0^t \frac{\omega}{1+t'^2} dt' = \omega \cdot \arctan(t') \Big|_{0=t'}^{t=t'} = \omega \cdot \text{atan}(t).$$

Therefore, $\vec{r}(t) = \left((1+t^2)\cos(\omega \,\text{atan}(t)), ((1+t^2)\sin(\omega \cdot \text{atan}(t))\right)$.

(b) $\theta = \omega \cdot \text{atan}(t) \Rightarrow t = \tan\left(\frac{\theta}{\omega}\right)$

$$\Rightarrow r(t) = 1+t^2 = 1 + \tan^2\left(\frac{\theta}{\omega}\right) \text{ "}=\text{" } r(\theta)$$

(c) We want there to be a solution to the equation $\theta = 2\pi$.

$\theta = \omega \cdot \text{atan}(t) < \omega \cdot \frac{\pi}{2}$, with $\lim\limits_{t\to\infty}(\theta) = \omega \cdot \frac{\pi}{2}$.

Thus, ω needs to be strictly larger than 4.