Math 14
Winter 2009
Monday, February 9

For these problems, we suppose that D is a y-simple region in the xy-plane. That is, D can be described by the inequalities

$$a \leq x \leq b$$

$$g(x) \leq y \leq h(x)$$

where g and h are continuous functions. We further assume that g and h are differentiable. We also let γ be the boundary of D, oriented counterclockwise, and let $P(x, y)$ be a continuously differentiable function from $\mathbb{R}^2$ to $\mathbb{R}$.

(1.) Sketch such a region D.

(2.) Show that

$$\iint_D \frac{\partial P}{\partial y}\, dA = \int_a^b P(x,\, h(x)) - P(x,\, g(x))\, dx.$$

(3.) Compare

$$\iint_D \frac{\partial P}{\partial y}\, dA$$

$$\text{to} \qquad \int_\gamma \langle P, 0\rangle \cdot \vec{T}\, ds \qquad \text{and to} \qquad \int_\gamma \langle 0, P\rangle \cdot \vec{n}\, ds.$$