

Math 14  
Supplementary Problems due Sept 29, 2003

1. In class we learned to define matrix multiplication and at the same time proved that the composition of two linear functions is linear. A function from  $R^n$  to  $R^m$  is called *affine* if it can be written as  $F(X) = MX + B$ , where  $M$  is a matrix,  $X$  is a column vector, and  $B$  is a column vector. How many entries does  $X$  have? How many entries does  $B$  have? How many rows does  $M$  have? How many columns does  $M$  have? Decide whether or not the composition of two affine functions is affine, and explain why you are correct. Note: You don't have to figure out the definition of matrix multiplication all over again!!!
2. Is it possible to find two three-by-three matrices  $A$  and  $M$  whose product is the all zeros matrix even though neither of  $A$  nor  $M$  are all-zero matrices? If so find two such matrices. If not explain why not.
3. Is it possible to two two-by-two matrices, all of whose entries are nonzero, such that the product of the two matrices is the all zeros matrix? If so, find two such matrices. If not, explain why not.
4. In Problem 14 from the book, you showed that if you multiply  $I_n$  times an  $n$  by  $n$  matrix  $A$ , on either side of  $A$ , you get  $A$  back, no matter what  $A$  is. Is there some other matrix  $J_n$  so that when you multiply any  $n$  by  $n$  matrix  $A$  by  $J_n$ , on either side of  $A$ , you get  $A$  back again. Either find such a  $J_n$  or explain why there is none.