Math 13, Winter 2018

Homework set 8, due Wed Feb 28

Please show your work. No credit is given for solutions without justification.

(1) Let $\mathcal{D}$ be a region in the plane with boundary curve $\mathcal{C}$ (oriented counter-clockwise). For which of the vector fields below (there may be one or more) is $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r} = \text{Area}\,(\mathcal{D})$?

(a) $\mathbf{F} = \langle x, 0 \rangle$
(b) $\mathbf{F} = \langle 0, x \rangle$
(c) $\mathbf{F} = \langle y, 2x \rangle$
(d) $\mathbf{F} = \langle \frac{1}{2}x, \frac{1}{2}y \rangle$

(2) Calculate the line integral $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ for the vector field

$$\mathbf{F} = \langle y - \frac{y}{x^2 + y^2}, x + \frac{x}{x^2 + y^2} \rangle$$

and the circle $\mathcal{C}$ with radius 5 and center $(1, 2)$.

Hint. Use Green's Theorem.

(3) Let $\mathbf{F}$ be a vector field in space with $\text{curl}\,(\mathbf{F}) = \langle x, y, -2z \rangle$. Let $\mathcal{C}_1$ be the parametrized circle $\mathbf{r}(t) = \langle \cos t, \sin t, 1 \rangle$ with $0 \leq t \leq 2\pi$, and $\mathcal{C}_2$ the parametrized circle $\mathbf{r}(t) = \langle \cos t, \sin t, 2 \rangle$ with $0 \leq t \leq 2\pi$, both oriented counter-clockwise when seen from above.

If we know that $\int_{\mathcal{C}_1} \mathbf{F} \cdot d\mathbf{r} = -2\pi$, find the value of $\int_{\mathcal{C}_2} \mathbf{F} \cdot d\mathbf{r}$ using Stokes' Theorem.