Homework set 5, due Wed Feb 7

Please show your work. No credit is given for solutions without justification.

(1) Let $\mathbf{F} = \langle xz + y, x, x^2 \rangle$.
  (a) Calculate $\operatorname{div} \mathbf{F}$ and $\operatorname{curl} \mathbf{F}$.
  (b) Is the vector field $\mathbf{F}$ *conservative*? If your answer is "No", explain why not. If your answer is "Yes", find a potential function by inspection.
  (c) If $\mathbf{F}$ represent a flow, is the vector field *incompressible*?

(2) Let $\mathcal{C}$ be the piecewise linear path (a triangle) in the xy-plane from $(0,0)$ to $(2,0)$ to $(0,3)$ to $(0,0)$. Evaluate the scalar line integral $\int_{\mathcal{C}} x^2 + y^2 \, ds$.

(3) Calculate the arc length of the parametrized curve $\mathbf{r}(t) = \langle e^t, t\sqrt{2}, e^{-t} \rangle$ with $0 \le t \le 2$.

   Hint. When you calculate $\|\mathbf{r}(t)\|$, try to make use of the formula $\sqrt{a^2 + 2ab + b^2} = |a+b|$.