Math 13, Winter 2018

Homework set 4, due Wed Jan 31

Please show your work. No credit is given for solutions without justification.

(1) Let $\mathcal{D}$ be the parallellogram in the xy-plane with vertices $(0,0)$, $(2,1)$, $(1,3)$ and $(3,4)$. Evaluate the integral $\iint_{\mathcal{D}} x+y\,dA$ by applying a linear change of variables that transforms $\mathcal{D}$ into the square $[0,1]\times[0,1]$.

(2) Do Exercise 32 from section 15.6.

(3) In section 15.4 you learned that in spherical coordinates, $dV = \rho^2 \sin\phi\, d\rho\, d\theta\, d\phi$. In section 15.6 it is explained how to find the Jacobian for an arbitrary change of variables for a triple integral,

$$dx\,dy\,dz = \left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right| du\,dv\,dw$$

(See the last page of 15.6.) Use this general formula to calculate the Jacobian for the change of variables from Cartesian coordinates (x,y,z) to spherical coordinates (ρ,θ,ϕ), and show that it is indeed $\rho^2\sin\phi$.