Math 13, Winter 2017

Homework set 9, due Wed Mar 8

Please show your work. No credit is given for solutions without justification.

(1) Use the surface integral in Stokes' theorem to calculate the circulation of the field $\mathbf{F}$ around the curve $\mathcal{C}$ in the indicated direction

$$\mathbf{F} = x^2y^3\mathbf{i} + \mathbf{j} + z\mathbf{k}$$

$\mathcal{C}$ is the boundary of the hemisphere $x^2 + y^2 + z^2 = 16$, $z \geq 0$, counterclockwise when viewed from above.

(2) Use the Divergence theorem to evaluate the surface integral $\int\int_S F \cdot d\mathcal{S}$ where $F(x, y, z) = (x + y, z, x)$ and $\mathcal{S}$ is the surface of the hemisphere $x^2 + y^2 + z^2 = 1$ with $z > 0$ and n is the outward normal to $\mathcal{S}$.

(3) Consider the region $\mathcal{R}$ bounded by the parabolas $y = x^2$ and $x = y^2$. Let $\mathcal{C}$ be the boundary of $\mathcal{R}$ oriented counterclockwise. Use Green's Theorem to evaluate

$$\int_{\mathcal{C}} (y + e^{\sqrt{x}})dx + (2x + \cos(y^2))dy$$