Math 13, Winter 2017

Homework set 1, due Wed Jan 11

Please show your work. No credit is given for solutions without justification.

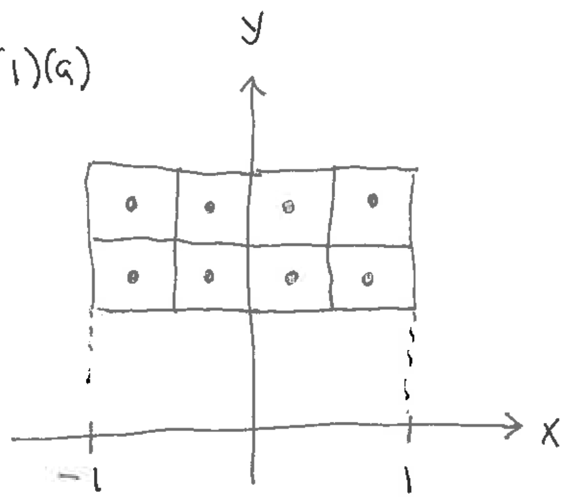
(1) (a) Calculate the value of a Riemann sum for the integral $\int_1^2 \int_{-1}^1 xy^2 \, dx \, dy$. Use a regular partition of the rectangle, with $\Delta x = \frac{1}{2}$ and $\Delta y = \frac{1}{2}$. Choose midpoints as sample points of the subrectangles.

(b) Find the exact value of the integral.

(2) Calculate the volume of the solid region below the paraboloid $z = 5 - x^2 - y^2$ and above the rectangle in the xy-plane with $0 \le x \le 2$ and $0 \le y \le 1$.

(3) Evaluate the double integral $\iint_{\mathcal{R}} xe^{xy} \, dA$, with $\mathcal{R} = [0,1] \times [0,1]$.

(1)(a)



sample points

X	Y	XY^2
$-\frac{3}{4}$	$\frac{5}{4}$	$-\frac{125}{16}$
$-\frac{1}{4}$	$\frac{5}{4}$	$-\frac{25}{16}$
$\frac{1}{4}$	$\frac{5}{4}$	$+\frac{25}{16}$
$\frac{3}{4}$	$\frac{5}{4}$	$+\frac{125}{16}$
$-\frac{3}{4}$	$\frac{7}{4}$	$-\frac{147}{16}$
$-\frac{1}{4}$	$\frac{7}{4}$	$-\frac{7}{16}$
$\frac{1}{4}$	$\frac{7}{4}$	$+\frac{7}{16}$
$\frac{3}{4}$	$\frac{7}{4}$	$+\frac{147}{16}$

cancels etc...

cancels

Total: 0

Since it is a regular partition

$$\Delta A = \Delta X \Delta Y = \frac{1}{4}$$

for all subrectangles.

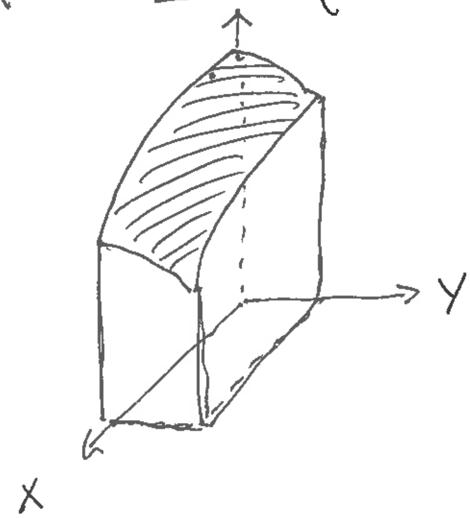
The Riemann Sum is

$$\sum_{i=1}^{4} \sum_{j=1}^{2} x_i y_j^2 \cdot \Delta A = 0 \cdot \frac{1}{4} = \boxed{0}$$

(1)(b) $\displaystyle\int_1^2 \left(\int_{-1}^1 xy^2\,dx\right) dy = \int_1^2 \left[\tfrac{1}{2}x^2y^2\right]_{x=-1}^{x=1} dy$

$\displaystyle = \int_1^2 \left(\tfrac{1}{2}y^2 - \tfrac{1}{2}y^2\right) dy = \int_1^2 0\,dy = \boxed{0.}$

(2) Sketch (not necessary)



$\displaystyle \text{Volume} = \int_0^2 \int_0^1 5 - x^2 - y^2\,dy\,dx$

$\displaystyle = \int_0^2 \left[5y - x^2y - \tfrac{1}{3}y^3\right]_{y=0}^{1} dx$

$\displaystyle = \int_0^2 5 - x^2 - \tfrac{1}{3}\,dx$

$\displaystyle = \left[\tfrac{14}{3}x - \tfrac{1}{3}x^3\right]_0^2$

$\displaystyle = \frac{28}{3} - \frac{8}{3} = \boxed{\frac{20}{3}}$

(3) $\displaystyle\int_0^1\int_0^1 xe^{xy}\,dy\,dx$ ← in the order $dx\,dy$ this integral is much harder.

First. $\displaystyle\int_0^1 xe^{xy}\,dy = \left[e^{xy}\right]_{y=0}^{1} = e^x - e^0 = e^x - 1.$

Then: $\displaystyle\int_0^1 e^x - 1\,dx = \left[e^x - x\right]_0^1 = (e-1) - (e^0 - 0)$

$= \boxed{e-2}$