Multiple Choice. *Each correct answer is worth 6 points. No points are subtracted for incorrect answers. You do not have to show your work for multiple choice questions. Extra paper is provided at the back of this booklet to do your calculations.*

(1) Evaluate the double integral $\iint_D e^{x^2+y^2}\,dA$ where D is the disk $x^2+y^2 \le 1$.

(A) πe (B) $2\pi e$ (C) $\pi e - \pi$ (D) $2\pi e - \pi$ (E) $\pi e - 2\pi$ (F) $2\pi e - 2\pi$

(2) Let $\mathcal{D}$ be the triangle in the xy-plane with vertices $(0,0)$, $(2,2)$ and $(4,2)$. Evaluate the integral $\iint_{\mathcal{D}} e^{x-y}dA$.

(A) e^2+2 (B) e^2+1 (C) e^2 (D) e^2-1 (E) e^2-2 (F) e^2-3

(3) Evaluate the double integral $\iint_D y\cos(xy)\,dA$ where D is the region in the xy-plane bounded by the hyperbola $xy=2$ and the lines $x=1$ and $y=1$.

(A) $\sin 2 - \sin 1$ (B) $\cos 2 - \cos 1$ (C) $\cos 2 + \sin 2 - \cos 1 - \sin 1$

(D) $\cos 2 + \sin 2 - \cos 1$ (E) $\cos 1 + \sin 1 - \sin 2$ (F) $\cos 2 - \sin 1$

(4) Find the volume of the solid region below the plane $z=3$ and above the cone $z=3\sqrt{x^2+y^2}$.

(A) π (B) 2π (C) 3π (D) 6π (E) 9π (F) 12π

(5) Find the average distance to the origin $(0,0,0)$ for all points (x,y,z) in the sphere $x^2+y^2+z^2=1$. The volume of a sphere of radius R is $\frac{4}{3}\pi R^3$.

(A) $\frac{1}{4}$ (B) $\frac{3}{8}$ (C) $\frac{1}{2}$ (D) $\frac{5}{8}$ (E) $\frac{3}{4}$ (F) $\frac{7}{8}$

1.) cyl. coords: $\displaystyle\int_{\theta=0}^{2\pi}\int_{r=0}^{1} r\cdot e^{r^2}\,dr\,d\theta = \frac{1}{2}\int_{\theta=0}^{2\pi}\left(e^{r^2}\Big|_{r=0}^{1}\right)d\theta = \underline{\pi e - \pi}$

2.) $\displaystyle\int_{y=0}^{2}\int_{x=y}^{2y} e^{x-y}\,dx\,dy = \underline{e^2-3}$

3.) $\displaystyle\int_{y=1}^{2}\int_{x=1}^{2/y} y\cos(xy)\,dx\,dy = \int_{y=1}^{2}\left(\sin(xy)\Big|_{x=1}^{2/y}\right)dy = \underline{\cos 2 + \sin 2 - \cos 1}$

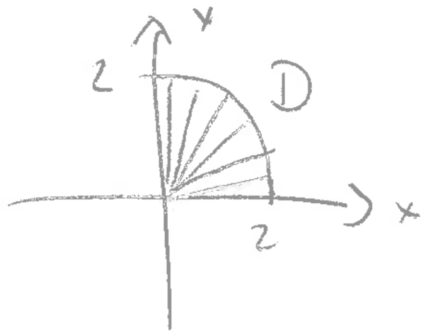
4.) cyl. coords: $\displaystyle\int_{\theta=0}^{2\pi}\int_{r=0}^{1}\int_{z=3r}^{3} 1\cdot r\,dz\,dr\,d\theta = \underline{\pi}$

5.) $\displaystyle\bar{f} = \frac{1}{4/3\,\pi}\cdot\int_{\theta=0}^{2\pi}\int_{\phi=0}^{\pi}\int_{\rho=0}^{1}\rho\cdot\rho^2\sin\phi\,d\rho\,d\phi\,d\theta = \underline{\frac{3}{4}}$

Free Response. *Each free response question is worth 10 points. On each question you must show your work. No credit is given for solutions without supporting calculations. You will get partial credit for partially correct answers.*

(6) What is the average value of the function $f(x, y) = x$ for the part of the disk $x^2 + y^2 \leq 4$ in the first quadrant (i.e. with $x \geq 0$ and $y \geq 0$)?

In polar coords:



$$D = \begin{cases} 0 \leq \varphi \leq \frac{\pi}{2} \\ 0 \leq r \leq 2 \end{cases}$$

$$f(x,y) = x = r \cdot \cos\varphi$$

1.) $$\iint_D f(x,y)\, dA = \int_{\varphi=0}^{\frac{\pi}{2}} \int_{r=0}^{2} r \cdot \cos\varphi \cdot r \, dr \, d\varphi$$

$$= \int_{\varphi=0}^{\frac{\pi}{2}} \cos\varphi \cdot \left(\frac{r^3}{3} \Big|_0^2 \right) d\varphi$$

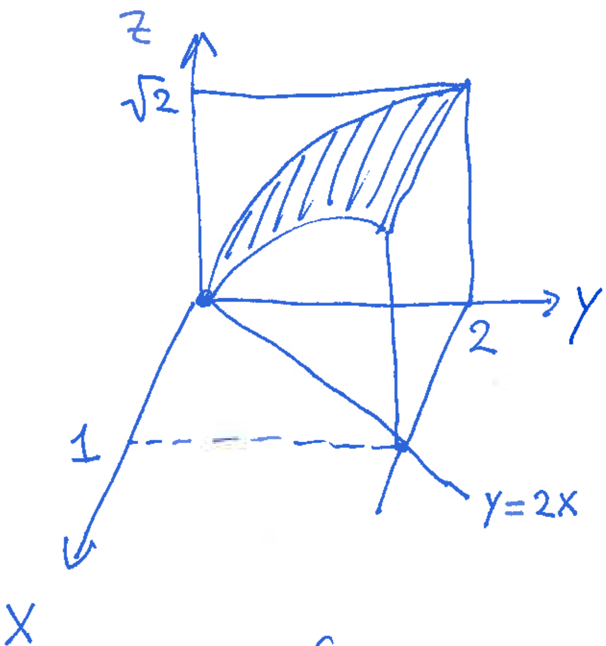
$$= \frac{8}{3} \int_{\varphi=0}^{\frac{\pi}{2}} \cos\varphi \, d\varphi = \frac{8}{3} \sin(\varphi) \Big|_{\varphi=0}^{\frac{\pi}{2}} = \frac{8}{3}$$

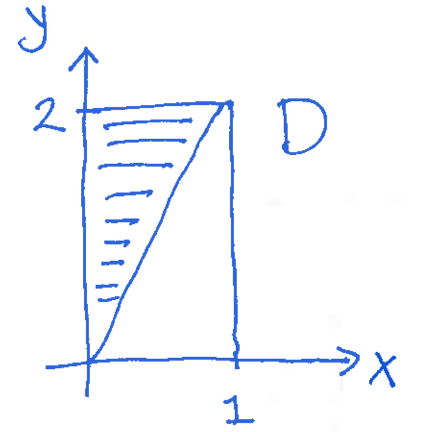
2.) $$\text{area}(D) = \iint_D 1 \, dA = \int_{r=0}^{2} \int_{\varphi=0}^{\frac{\pi}{2}} r \, d\varphi \, dr$$

$$= \frac{\pi}{2} \cdot \left(\frac{r^2}{2} \Big|_{r=0}^{2} \right) = \pi$$

1.) / 2.) $\Longrightarrow$ $$\bar{f} = \frac{1}{\text{area}(D)} \iint_D f(x,y) \, dA = \underline{\underline{\frac{8}{3\pi}}}$$

(7) Find the volume of the solid bounded by the planes $x = 0$, $y = 2$, $z = 0$, $y = 2x$ and the surface $y = z^2$.





projection onto xy plane

$$D \begin{cases} 0 \le x \le 1 \\ 2x \le y \le 2 \end{cases}$$

$$0 \le z \le \sqrt{y} \leftarrow \text{from } y = z^2$$

$$\text{Volume} = \iiint 1\, dA = \int_0^1 \int_{2x}^{2} \int_0^{\sqrt{y}} 1\, dz\, dy\, dx$$

- $\displaystyle \int_0^{\sqrt{y}} 1\, dz = \left[z \right]_{z=0}^{\sqrt{y}} = \sqrt{y} = y^{1/2}$

- $\displaystyle \int_{2x}^{2} y^{1/2} dy = \left[\frac{2}{3} y^{3/2} \right]_{y=2x}^{2} = \frac{2}{3} 2^{3/2} - \frac{2}{3} (2x)^{3/2} = \frac{4\sqrt{2}}{3} \left(1 - x^{3/2} \right)$

- $\displaystyle \int_0^1 \frac{4\sqrt{2}}{3} - \frac{4\sqrt{2}}{3} x^{\frac{3}{2}} dx = \frac{4\sqrt{2}}{3} \cdot \left[x - \frac{2}{5} x^{\frac{5}{2}} \right]_{x=0}^{1}$

$$= \frac{4\sqrt{2}}{3} \cdot \left(1 - \frac{2}{5} \right) = \frac{4\sqrt{2}}{3} \cdot \frac{3}{5} = \boxed{\frac{4\sqrt{2}}{5}}$$

(8) Find the volume of the solid region inside the sphere $x^2 + y^2 + z^2 \leq 4$ and above the cone $z = \sqrt{3x^2 + 3y^2}$.

$$\rho \cos\varphi = \sqrt{3}\sqrt{\rho^2 \sin^2\varphi \cos^2\theta + \rho^2 \sin^2\varphi \sin^2\theta}$$

$$\Rightarrow \rho\cos\varphi = \sqrt{3}\,\rho\sin\varphi \Rightarrow \tan\varphi = \frac{1}{\sqrt{3}} \Rightarrow \varphi = \frac{\pi}{6}$$

$$W = \{0 \leq \theta \leq 2\pi,\ 0 \leq \varphi \leq \pi/6,\ 0 \leq \rho \leq 2\}$$

$$\text{Volume}(W) = \int_0^{2\pi}\int_0^{\pi/6}\int_0^{2} \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/6} \frac{\rho^3}{3}\Big|_0^2 \sin\varphi \, d\varphi \, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/6} \frac{8}{3} \sin\varphi \, d\varphi \, d\theta$$

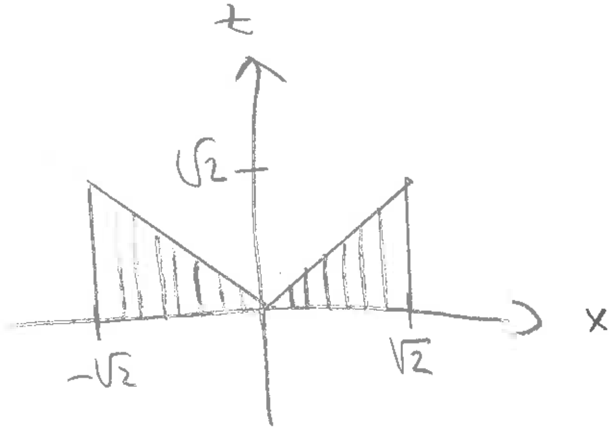
$$\int_0^{2\pi} \frac{8}{3}(-\cos\varphi)\Big|_0^{\pi/6} d\theta$$

$$= -\frac{8}{3}\int_0^{2\pi}[\cos(\pi/6) - \cos(0)]\, d\theta$$

$$= -\frac{8}{3}(2\pi)\left(\frac{\sqrt{3}}{2} - 1\right)$$

$$= \frac{16\pi}{3}\left(1 - \frac{\sqrt{3}}{2}\right)$$

(9) Let W be the region within the cylinder $x^2+y^2=2$, above the xy-plane $z=0$, and below the cone $z=\sqrt{x^2+y^2}$. Calculate the integral $\iiint_W f(x,y,z)\,dV$ for the function $f(x,y,z)=x^2+y^2+z^2$.



$$W = \begin{cases} 0 \le \theta \le 2\pi \\ 0 \le r \le \sqrt{2} \\ 0 \le z \le r \end{cases}$$

in cylindrical coordinates.

- $f(x,y,z) = x^2+y^2+z^2 = r^2+z^2$

in cylindrical coordinates.

$$\iiint_W f(x,y,z)\,dV = \int_{\theta=0}^{2\pi}\int_{r=0}^{\sqrt{2}}\int_{z=0}^{r} (r^2+z^2)\cdot r\,dz\,dr\,d\theta$$

1.) $\displaystyle\int_{z=0}^{r} r^3+rz^2\,dz = r^3z+\frac{z^3}{3}r\Big|_{z=0}^{r} = \frac{4}{3}r^4$

2.) $\displaystyle\int_{r=0}^{\sqrt{2}} \frac{4}{3}r^4\,dr = \frac{4}{3}\cdot\frac{r^5}{5}\Big|_{r=0}^{\sqrt{2}} = \frac{4}{15}\cdot 2^{5/2}$

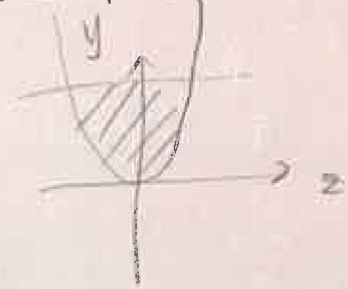
3.) $\displaystyle\int_{\theta=0}^{2\pi} \frac{4}{15}\cdot 2^{5/2}\,d\theta = 2\pi\cdot\frac{4}{15}\cdot 2^{5/2}$

$$= \frac{32}{15}\pi\cdot\sqrt{2}$$

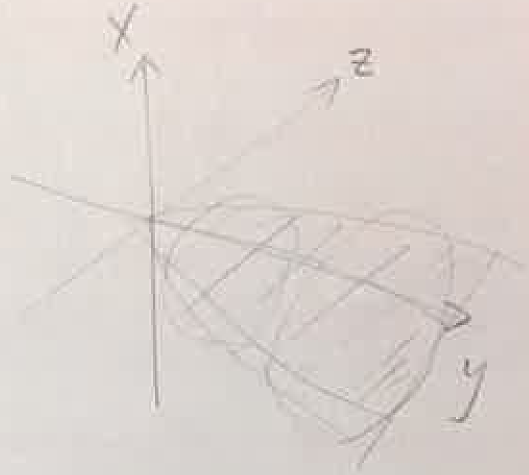
$x^2 \le y - z^2$

$x^2+z^2 \le y$

yz-projection



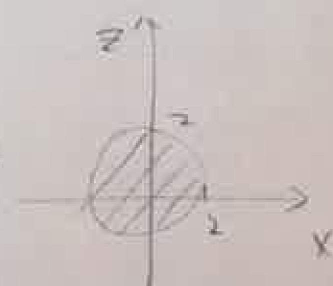
$x^2+z^2 \le y \le 4$



$x^2+z^2 \le 4$

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} dz\,dx$$

xz-projection



$$\boxed{\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+z^2}^{4} dy\,dz\,dx}$$