HOMEWORK 5 (DUE WEDNESDAY FEBRUARY 10)

(1) (Problem 58 in §16.2) Note that a curve $\mathcal{C}$ in polar form $r = f(\theta)$ is parametrized by $\boldsymbol{r} = (f(\theta)\cos\theta, f(\theta)\sin\theta)$ becaue the x- and y-coordinates are given by $x = r\cos\theta$ and $y = r\sin\theta$.
   (a) Show that $\|\boldsymbol{r}'(\theta)\| = \sqrt{f(\theta)^2 + f'(\theta)^2}$.
   (b) Evaluate $\int_{\mathcal{C}} (x-y)^2\, ds$, where $\mathcal{C}$ is the semicircle with polar equation $r = 2\cos\theta,\ 0 \le \theta \le \frac{\pi}{2}$.

(2) (Problem 46 in §16.1) Find potential functions for $\boldsymbol{F} = \dfrac{\boldsymbol{e_r}}{r^3}$ and $\boldsymbol{G} = \dfrac{\boldsymbol{e_r}}{r^4}$ in $\mathbb{R}^3$.

(3) (Problem 69 in §16.2) Let $\boldsymbol{F}(x,y) = \langle x, 0\rangle$. Prove that if $\mathcal{C}$ is any path from (a,b) to (c,d), then

$$\int_{\mathcal{C}} \boldsymbol{F}\cdot d\boldsymbol{r} = \tfrac{1}{2}(c^2 - a^2).$$

(4) (Problem 30 in §16.3) The vector field $\boldsymbol{F} = \left\langle \dfrac{x}{x^2+y^2}, \dfrac{y}{x^2+y^2} \right\rangle$ is defined on the domain $\mathcal{D} = \{(x,y) \neq (0,0)\}$.
   (a) Is $\mathcal{D}$ simply connected?
   (b) Show that $\boldsymbol{F}$ satisfies the cross-partial condition. Does this guarantee that $\boldsymbol{F}$ is conservative?
   (c) Show that $\boldsymbol{F}$ is conservative on $\mathcal{D}$ by finding a potential function.
   (d) Do these results contradict Theorem 4?

(5) (Problem 50 in §16.2) Calculate $V(P)$ at the origin $P = (0,0)$ if the electric charge is distributed along $y = x^{-1}$ for $\frac{1}{2} \le x \le 2$ with charge density $\delta(x,y) = x^3 y$.

(6) (Problem 28 in §16.3) An electron at rest at $P = (5,3,7)$ moves along a path ending at $Q = (1,1,1)$ under the influence of the electric field (in newtons per coulomb)

$$\boldsymbol{F}(x,y,z) = 400(x^2+z^2)^{-1}\langle x, 0, z\rangle.$$

   (a) Find a potential function for $\boldsymbol{F}$.
   (b) What is the electron's speed at the point Q? (Use conservation of energy and the value $q_e/m_e = -1.76 \times 10^{11}\,\mathrm{C/kg}$, where q_e and m_e are the charge and mass on the electron, respectively.