HOMEWORK 2 (DUE WEDNESDAY JANUARY 20)

(1) (Problem 32 in §15.5) Calculate I_x and I_y for the half-disk $x^2 + y^2 \leq R^2$, $x \geq 0$ (in meters), with total mass M kg and uniform mass density.

(2) (Problem 24 in §15.3) Describe (and sketch) the domain of integration and evaluate:

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2-y^2}} xy \, dz \, dy \, dx$$

(3) (Problem 60 in §15.4) Recall that the improper integral $\int_0^1 x^{-a} \, dx$ converges if and only if $a < 1$. For which values of a does $\iint_{\mathcal{D}} r^{-a} \, dA$ converge when $r = \sqrt{x^2 + y^2}$ and $\mathcal{D}$ is the unit disk $x^2 + y^2 \leq 1$?

(4) (Problem 41 in §15.4) Use cylindrical coordinates to find the volume of a sphere of radius a from which a central cylinder of radius b has been removed, where $0 < b < a$.

(5) (Problem 48 in §15.4) Use spherical coordinates to calculate

$$\iiint_{\mathcal{W}} 1 \, dV,$$

where $\mathcal{W}$ is the region

$$x^2 + y^2 + z^2 \leq 4z, \quad z \geq \sqrt{x^2 + y^2}.$$