Math 13, Homework #9

Due Monday, March 7, 2016

1. (17.3.27) The electric field due to a unit electric dipole oriented in the $\vec{k}$-direction is $\vec{E} = \nabla(z/r^3)$, where $r = \sqrt{x^2+y^2+z^2}$. Let $\vec{e_r} = r^{-1}\langle x, y, z\rangle$.

(a) Show that $\vec{E} = r^{-3}\vec{k} - 3zr^{-4}\vec{e_r}$.

(b) Calculate the flux of $\vec{E}$ through a sphere centered at the origin.

(c) Calculate $div(E)$.

(d) Can we use the Divergence Theorem to compute the flux of $\vec{E}$ through a sphere centered at the origin?

2. (17.2.18) Let $\vec{F} = \langle 0, -z, 1\rangle$. Let $\mathcal{S}$ be the spherical cap $x^2+y^2+z^2 \leq 1$, where $z \geq \frac{1}{2}$.

(a) Evaluate $\iint_{\mathcal{S}} \vec{F} \cdot d\vec{S}$ directly as a surface integral.

(b) Verify that $\vec{F} = curl(\vec{A})$, where $\vec{A} = \langle 0, x, xz\rangle$.

(c) Evaluate the flux of $\vec{F}$ through $\mathcal{S}$ again by using Stokes' Theorem.

3. (17.2.11) Let $\vec{F} = \langle 3y, -2x, 3y\rangle$, and $\mathcal{C}$ be the circle $x^2+y^2 = 9, z = 2$, oriented counterclockwise as viewed from above. Apply Stokes' Theorem to evaluate $\oint_{\mathcal{C}} \vec{F} \cdot d\vec{r}$ by finding the flux of $curl(\vec{F})$ across an appropriate surface.