Math 13, Homework #6

Due February 17, 2016

1. (17.1.8) Use Green's Theorem to evaluate the line integral given by

$$\oint_{\mathcal{C}} (\ln x + y)\, dx - x^2\, dy\,,$$

where $\mathcal{C}$ is the rectangle with vertices $(1,1),(3,1),(1,4),(3,4)$, oriented counterclockwise.

2. (17.1.19) Consider the curve in the xy-plane given by $x^3 + y^3 = 3xy$ (this curve is known as the *folium of Descartes*).

(a) Show that the curve has a parametrization in terms of $t = y/x$ given by

$$x = \frac{3t}{1+t^3}, \qquad y = \frac{3t^2}{1+t^3}\,, \qquad (-\infty < t < \infty) \quad (t \neq -1)\,.$$

(b) Show that

$$x\,dy - y\,dx = \frac{9t^2}{(1+t^3)^2}\,dt\,.$$

Hint: By the Quotient Rule, $x^2\, d(\frac{y}{x}) = x\,dy - y\,dx$.

(c) Find the area of the loop enclosed by the curve in the first quadrant. *Hint:* The limits of integration are 0 and ∞.

3. (17.1.21) The *centroid* of a domain $\mathcal{D}$ enclosed by a simple closed curve $\mathcal{C}$ is the point with coordinates $(\bar{x}, \bar{y}) = (M_y/M, M_x/M)$, where M is the area of $\mathcal{D}$ and the moments are defined by

$$M_x = \int\int_{\mathcal{D}} y\, dA\,, \qquad \text{and} \qquad M_y = \int\int_{\mathcal{D}} x\, dA\,.$$

Show that $M_x = \oint_{\mathcal{C}} xy\, dy$. Find a similar expression for M_y.

4. (16.1.30) Let the vector field $\vec{F}(x,y,z)$ be defined by

$$\vec{F} = < \frac{x}{x^2+y^2}, \frac{y}{x^2+y^2}, 0 > \,.$$

Calculate $div(\vec{F})$ and $curl(\vec{F})$.

5. (16.1.38) Find (by inspection) a potential function for

$$\vec{F} = < x, 0 >\,,$$

and show that

$$\vec{G} = < y, 0 >$$

is not conservative.