Math 13 Winter 2015 Your name: ____________________

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Problem set 7, due Wed Feb 25

Please show your work. No credit is given for solutions without justification.

(1) Find an equation of the tangent plane to the surface $\mathbf{r}(u,v) = \langle uv, u+v, u-v\rangle$ at the point $(1,2,0)$.

(2) Find the surface area of the part of the plane $2x+y+2z=8$ that lies inside the cylinder $x^2+y^2=2x$.

(3) Find the surface area of the part of the paraboloid $y = x^2+z^2$ that lies to the left of the plane $y=4$.

① at point $(1,2,0)$ $u=1,\ v=1$.

$\vec{r}_u(1,1) = (v, 1, 1)\big|_{\substack{u=1\\v=1}} = (1,1,1)$.

$\vec{r}_v(1,1) = (u, 1, -1)\big|_{\substack{u=1\\v=1}} = (1,1,-1)$

$\vec{r}_u \times \vec{r}_v\big|_{\substack{u=1\\v=1}} = (1,1,1)\times(1,1,-1) = (-2,2,0)$.

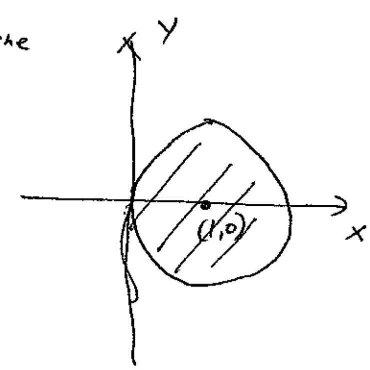
so the tangent plane at $(1,2,0)$ is:

$$(-2)(x-1) + 2(y-2) + 0(z-0) = 0$$

$$-2x+2+2y-4 = 0$$

$$\underline{y = x+1}.$$

② $x^2+y^2=2x$ is the circle $(x-1)^2+y^2=1$ on the xy-plane



$z = \frac{1}{2}(8-2x-y) = 4-x-\frac{1}{2}y$.

so $\vec{r}(x,y) = (x, y, 4-x-\frac{1}{2}y)$

$\vec{r}_x = (1, 0, -1)$.

$\vec{r}_y = (0, 1, -\frac{1}{2})$

$\vec{r}_x \times \vec{r}_y = (1, \frac{1}{2}, 1)$ and so $|\vec{r}_x\times\vec{r}_y| = \sqrt{1^2+(\frac{1}{2})^2+1^2} = \sqrt{\frac{9}{4}} = \frac{3}{2}$.

$A(S) = \iint_D \frac{3}{2}\,dA = \frac{3}{2}A(D) = \frac{3}{2}\pi$.

③ $\vec{r}(x,z) = (x,\ x^2+z^2,\ z)$

$\vec{r}_x = (1,\ 2x,\ 0)$

$\vec{r}_z = (0,\ 2z,\ 1)$

$\vec{r}_x \times \vec{r}_z = (2x,\ -1,\ 2z)$

$A(S) = \iint_D \sqrt{4x^2+1+4z^2}\, dA$ where D is the disk $x^2+z^2 \le 4$.

$= \int_0^{2\pi} \int_0^2 \sqrt{4r^2+1}\ r\,dr\,d\theta$

$= \int_0^{2\pi} \frac{1}{8} \int_0^2 (4r^2+1)^{1/2}\, d(4r^2+1)\ d\theta$

$= \int_0^{2\pi} \frac{1}{8} \cdot \frac{2}{3} \cdot (4r^2+1)^{3/2} \Big|_0^2\ d\theta$

$= 2\pi \cdot \frac{1}{8} \cdot \frac{2}{3} \cdot (17\sqrt{17} - 1)$

$= \frac{\pi}{6} (17\sqrt{17} - 1).$