Math 13 Winter 2015 Your name: ____________________

Instructor (please circle): □ Pierre Clare □ Mingzhong Cai □ Erik van Erp

Problem set 3, due Wed Jan 28

Please show your work. No credit is given for solutions without justification.

(1) Find the surface area of the part of the plane $3x + 4y + 5z = 12$ that is inside the first octant.

(2) Evaluate the triple integral $\iiint_E xyz\, dV$ where E is the solid tetrahedron bounded by the planes $x = 1$, $y = 0$, $z = 0$ and $-x + y + z = 0$.

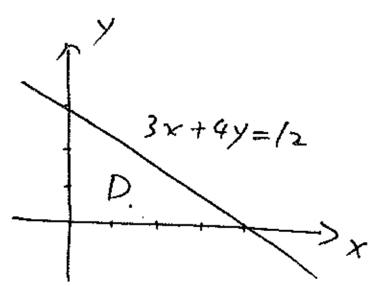
(3) Write an iterated integral that is equal to

$$\int_0^1 \int_y^1 \int_0^z f(x, y, z)\, dx\, dz\, dy$$

but change the order of integration from $\iiint \ldots dx\, dz\, dy$ to $\iiint \ldots dy\, dx\, dz$.

① $z = \frac{1}{5}(12 - 3x - 4y)$

$\frac{\partial z}{\partial x} = -\frac{3}{5}$, $\frac{\partial z}{\partial y} = -\frac{4}{5}$.



so $A(S) = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dA$

$= \iint_D \sqrt{1 + \left(-\frac{3}{5}\right)^2 + \left(-\frac{4}{5}\right)^2}\, dA$

$= \sqrt{2} \iint_D dA$

$= \sqrt{2} \cdot A(D)$

$= \sqrt{2} \cdot \frac{3 \cdot 4}{2}$

$= 6\sqrt{2}$.

(2) $\displaystyle\iiint_E xyz\, dV$

$$= \int_0^1 \int_0^x \int_0^{x-y} xyz \, dz\, dy\, dx$$

$$= \int_0^1 \int_0^x xy \cdot \frac{1}{2} z^2 \Big|_{z=0}^{z=x-y} dy\, dx$$

$$= \frac{1}{2} \int_0^1 \int_0^x xy(x^2 - 2xy + y^2)\, dy\, dx$$

$$= \frac{1}{2} \int_0^1 \int_0^x (x^3 y - 2x^2 y^2 + xy^3)\, dy\, dx$$

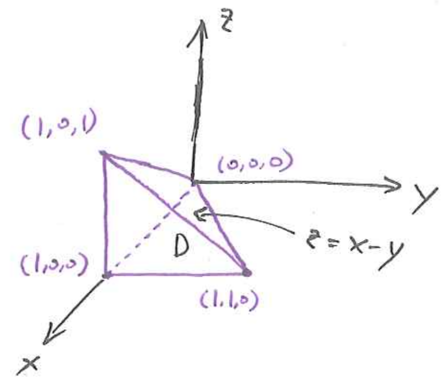
$$= \frac{1}{2} \int_0^1 \left(\frac{1}{2} x^3 y^2 - \frac{2}{3} x^2 y^3 + \frac{1}{4} x y^4 \right) \Big|_{y=0}^{y=x} dx$$

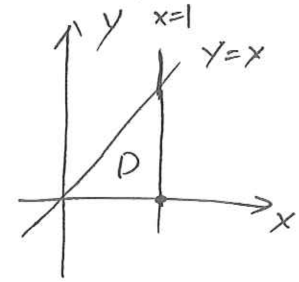
$$= \frac{1}{2} \int_0^1 \left(\frac{1}{2} x^5 - \frac{2}{3} x^5 + \frac{1}{4} x^5 \right) dx$$

$$= \frac{1}{2} \cdot \frac{1}{12} \int_0^1 x^5\, dx$$

$$= \frac{1}{2} \cdot \frac{1}{12} \cdot \frac{1}{6}$$

$$= \frac{1}{144}$$





(3) $\displaystyle\int_0^1 \int_y^1 \int_0^z f(x,y,z)\, dx\, dz\, dy$

$$= \int_0^1 \int_0^z \int_0^z f(x,y,z)\, dy\, dx\, dz$$