Worksheet Jan 10

(1) Define $g : \mathbf{R}^2 \to \mathbf{R}^2$ by $g(s,t) = (\ln(st), (s+t)^2)$ and $f : \mathbf{R}^2 \to \mathbf{R}^2$ by $f(x,y) = (e^{xy}, (x+1)^2 y.)$

(a) Find the derivative matrix $(f \circ g)'(1,1)$.

(b) Writing $(u,v) = f(x,y)$ and $(x,y) = g(s,t)$, read off each of the partials $\frac{\partial u}{\partial s}$, $\frac{\partial u}{\partial t}$, $\frac{\partial v}{\partial s}$ and $\frac{\partial v}{\partial t}$ at $(s,t) = (1,1)$ from your answer to part (a). (No computation needed!)

(c) Writing $v = (x+1)^2 y$ with $x = \ln(st)$ and $y = (s+t)^2$, compute $\frac{\partial v}{\partial s}$ at $(s,t) = (1,1)$ the way you learned in Math 8 (or BC calculus) and compare with your answer in (b).

(2) Let

$$f(x,y,z) = e^{2x}(yz+1)^2.$$

Find the maximum rate of increase of f at $(0,1,1)$ and the direction in which it occurs (specified by a unit vector).