Math 13: Written Homework #6.
Due Monday, February 18, 2013.

1. (§16.3, #29) Suppose that the vector field $\mathbf{F} = \langle P, Q, R \rangle$ is conservative and the P, Q and R have continuous first-order partial derivatives. Explain why

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}, \quad \frac{\partial P}{\partial z} = \frac{\partial R}{\partial x}, \quad \text{and} \quad \frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y}.$$

2. (§16.3, #14) Find a potential function $f(x, y)$ for $\mathbf{F} = \langle (1 + xy)e^{xy}, x^2 e^{xy} \rangle$, and evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{r},$$

where C is the curve parametrized by $\vec{r}(t) = \langle \cos t, 2 \sin t \rangle$ for $t \in [0, \pi/2]$.

3. (§16.3, #36a) Suppose that $\mathbf{F}$ is an inverse square field; that is,

$$\mathbf{F}(\mathbf{r}) = \frac{c\mathbf{r}}{|\mathbf{r}|^3},$$

where c is a constant and $\mathbf{r} = \langle x, y, z \rangle$. Find the work done is moving an object from point P_1 to P_2 in terms of the distances d_j of P_j to the origin.

4. (§16.4, #2) Verify Green's Theorem by evaluating the line integral below by (a) using Green's Theorem, and (b) by direct evaluation.

$$\int_C xy\, dx + x^3\, dy,$$

where C is the rectangle (oriented counterclockwise) with vertices $(0, 0)$, $(3, 0)$, $(3, 1)$ and $(0, 1)$.

5. Evaluate the line integral

$$\int_C (4x^2 y + e^{x^2})\, dx + (9xy^2 + \sin(y^2))\, dy$$

around the ellipse $4x^2 + 9y^2 = 36$ in a counterclockwise direction.

6. Compute

$$\int_C (e^{x^2}\,dx + dy),$$

where C is the semicircle $x^2 + y^2 = 1$ with $x \geq 0$ from $(0, -1)$ to $(0, 1)$.