Math 13: Written Homework #4.
Due Wednesday, February 6, 2013.

1. (§15.10, #19.) Use the transformation $x = u/v$ an $y = v$ to evaluate

$$\iint_R xy\, dA,$$

where R is the planar region in the first quadrant bounded by the lines $y = x$, $y = 3x$, and the hyperbolas $xy = 1$ and $xy = 3$.

2. (§15.10, #23.) Use an appropriate change of variables to evaluate

$$\iint_R \frac{x - 2y}{3x - y}\, dA,$$

where R is the parallelogram enclosed by the lines $x - 2y = 0$, $x - 2y = 4$, $3x - y = 1$ and $3x - y = 8$.

3. Let E be a solid region lying above the xy-plane and inside the ellipsoid

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1.$$

Assuming that E has constant density k, find the center of mass $(\bar{x}, \bar{y}, \bar{z})$ of E. You may assume that, by symmetry, $\bar{x} = 0 = \bar{y}$. (Suggestion: make a change of variables so that you can use spherical coordinates.)

4. (§14.3, #72.) Let $g(x, y, z) = \sqrt{1 + xz} + \sqrt{1 - xy}$. Compute g_{xyz}. (This problem is very easy if you use a different order of differentiation for each term.)

5. (§14.4, #42.) Suppose you need to know the equation of the tangent place to a surface S at the point $(2, 1, 3)$. You don't have an equation for S, but you know that the curves $\mathbf{r}(t) = \langle 2 + 3t, 1 - t^2, 3 - 4t + t^2 \rangle$ and $\mathbf{s}(u) = \langle 1 + u^2, 2u^3 - 1, 2u + 1 \rangle$ both lie on S. Find the equation of the tangent place at the point $(2, 1, 3)$.