MATH 13. Multivariable Calculus. Written Homework 1.

Due on Monday, 1/14/13.

1. Consider the following limit:

$$\lim_{(x,y)\to(0,0)} \frac{y^2 \sin(x^2)}{x^4 + y^4}.$$

   If it exists, find its value. Otherwise explain why it doesn't exits.

2. Evaluate

$$\lim_{n\to\infty} \sum_{k=1}^{n} \sqrt{\frac{k}{n^3}}$$

   by realizing it as a limit of Riemann sums.

3. #5 (in section 15.1) from page 1005 in our text.

4. If k is a constant function, $f(x, y) = k$, and $R = [a, b] \times [c, d]$, show that

$$\iint_R k \; dA = k(b - a)(d - c).$$

5. Show that

$$0 \le \iint_R \sin \pi x \cos \pi y \; dA \le \frac{1}{32},$$

   where $R = [0, \frac{1}{4}] \times [\frac{1}{4}, \frac{1}{2}]$ (Hint : Use the result of problem 4 and equation (9) on page 1005 of the text.)

6. In evaluating a double integral over a region D, a sum of iterated integrals was obtained as follows:

$$\iint_D f(x, y)dA = \int_0^2 \int_0^{\sqrt{y}} f(x, y)dxdy + \int_2^4 \int_{y-2}^{\sqrt{y}} f(x, y)dxdy.$$

   Sketch the region D and express the double integral as an iterated integral with reversed order of integration.