Math 13 Spring 2011

Multivariable Calculus

Exam II

Wednesday May 18, 6-8 PM

Your name (please print): _____

Instructor (circle one): Sutton, Yang

Instructions: This is a closed book, closed notes exam. **Use of calculators is not permitted**. You must justify all of your answers to receive credit, unless instructed otherwise in a given problem. On the multiple choice questions, only the answer you mark on the scantron form will be counted and justifications can be minimal.

You have **two hours** to work on all **15** problems. Please do all your work in this exam booklet.

The Honor Principle requires that you neither give nor receive any aid on this exam.

FERPA Waiver: By my signature I relinquish my FERPA rights in the following context: My exam may be returned en masse with others present in the classroom. I acknowledge that I understand my score may be visible to others. If I choose not to relinquish my FERPA rights, I understand that I will have to present my student ID at my instructor's office to retrieve my exam.

Signature:_____

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Your name (please print): _____

Problem	Points	Score
MC	40	
11	12	
12	12	
13	12	
14	12	
15	12	
Total	**100**	

MULTIPLE CHOICE QUESTIONS

In Questions 1-3 you will be given some information about a differentiable vector field $\mathbf{F}$ defined on some region D. Based on the information given, can you determine whether $\mathbf{F}$ is conservative, not conservative, or is there not enough information to decide?

(1) $\mathbf{F}$ is defined on $D = \mathbb{R}^2$, C is the closed curve $x^2 + y^2 = 4$, and $\int_C \mathbf{F} \cdot d\mathbf{r} \neq 0$.
(a): $\mathbf{F}$ is conservative
(b): $\mathbf{F}$ is *not* conservative
(c): There is not enough information

(2) $\mathbf{F}$ is defined on $D = \mathbb{R}^2$, C is the closed curve $x^2 + y^2 = 4$, and $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$.
(a): $\mathbf{F}$ is conservative
(b): $\mathbf{F}$ is *not* conservative
(c): There is not enough information

(3) $\mathbf{F}(x, y) = P(x, y)\,\mathbf{i} + Q(x, y)\,\mathbf{j}$ is defined D where D is the interior of the triangle with vertices $(-1, 2), (2, 4), (3, 6)$, and $P_y = Q_x$.
(a): $\mathbf{F}$ is conservative
(b): $\mathbf{F}$ is *not* conservative
(c): There is not enough information

(4) If $\mathbf{F}$ is a conservative vector field on a region $D \subset \mathbb{R}^2$, then
- **(a):** $\int_C \mathbf{F} \cdot d\mathbf{r}$ depends only on the endpoints of a curve C in D.
- **(b):** $\int_C \mathbf{F} \cdot d\mathbf{r}$ is zero for any closed curve C in D.
- **(c):** $\mathbf{F} = \nabla f$ for some differentiable function f on D.
- **(d):** All of the above.

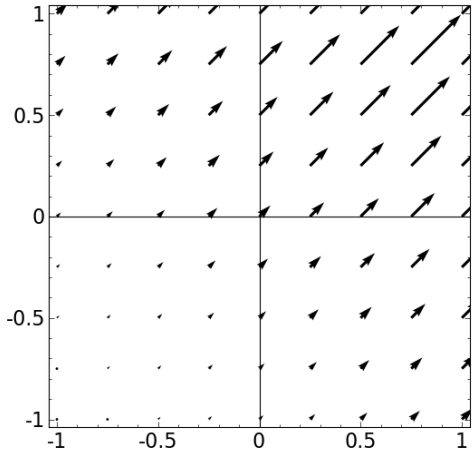
(5) Let $\mathbf{F} = \langle e^x, y + e^z, 2x + y \rangle$. Compute $\nabla \times \mathbf{F}$.
- **(a):** $\langle e^x, 1, 0 \rangle$
- **(b):** $\langle 1 - e^z, -2, 0 \rangle$
- **(c):** $\langle e^x, -1, 0 \rangle$
- **(d):** $\langle 1 - e^z, 2, 0 \rangle$

(6) Let $\mathbf{F}(x, y, z) = \langle x + \sin y, y - \sin z, z \rangle$. Then div $\mathbf{F}$ is equal to
(a): 3
(b): 0
(c): $\langle \cos(z), 0, -\cos(y) \rangle$
(d): $\langle 1, 1, 1 \rangle$

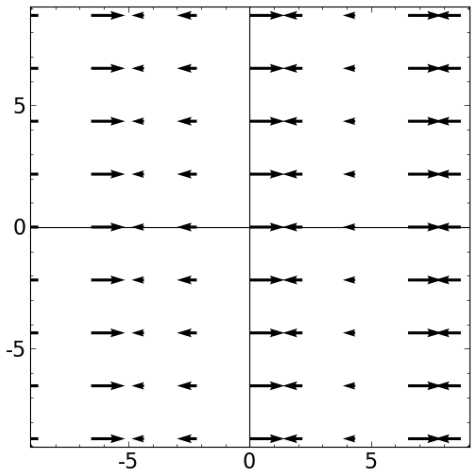
In questions 7-10 please match each of the following functions with the plot of its gradient vector field. Each plot is labelled by the letter located above it.

(7) $f(x, y) = x^2 + y^2$
(8) $f(x, y) = \sin x$
(9) $f(x, y) = e^{x+y}$
(10) $f(x, y) = \frac{1}{xy}$

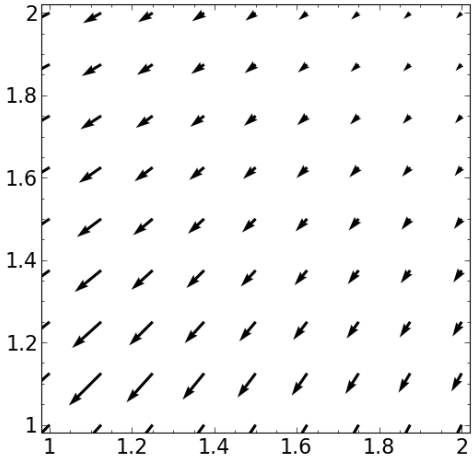
(A)



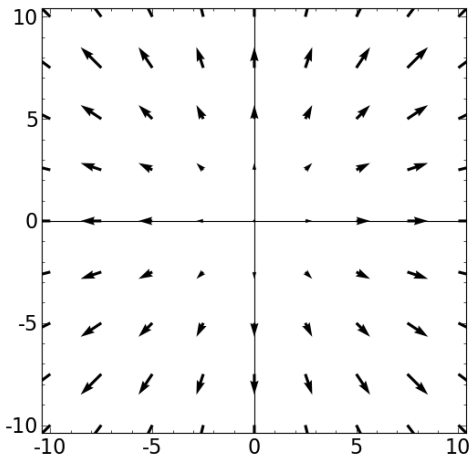
(B)



(C)



(D)



NON-MULTIPLE CHOICE QUESTIONS

(11) Find a function $f(x, y, z)$ such that $f(0, 1, 0) = 5$ and $\nabla f(x, y, z) = (2xe^y + z^2)\,\mathbf{i} + (x^2e^y + \cos z)\,\mathbf{j} + (2xz - y\sin z)\,\mathbf{k}$.
Please show your work.

(12) Let C be the boundary of the square with vertices $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$ oriented counterclockwise and let $\mathbf{F} = 2xy\,\mathbf{i} + (3x^2 + \cos(e^y))\,\mathbf{j}$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

(13) Let $\mathbf{F} = 4x^3y^3\,\mathbf{i} + 3x^4y^2\,\mathbf{j}$. Find the value of $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve with initial point $(1, 2)$ and terminal point $(-2, 1)$ depicted in the figure below.

(14) Let E be the region in $\mathbb{R}^3$ defined by the inequalities $1 \leq x^2 + y^2 + z^2 \leq 4$. Find the average value of the function $f(x, y, z) = z^2$ on E. Please simplify your answer as much as possible.

(15) Let E be the region in $\mathbb{R}^3$ defined by the inequalities

$$\begin{aligned} 4 \leq\ x^2 + y^2 &\leq 9 \\ x, y &\geq 0 \\ -2 \leq\ z &\leq 3. \end{aligned}$$

Evaluate

$$\int\int\int_E xyz\, dV.$$