Math 13 Spring 2011

Multivariable Calculus

Exam I

Wednesday April 20, 6:00-8:00 PM

Your name (please print): ______________________________

Instructor (circle one): Sutton, Yang

Instructions: This is a closed book, closed notes exam. **Use of calculators is not permitted**. You must justify all of your answers to receive credit, unless instructed otherwise in a given problem. On the multiple choice questions, only the answer you mark on the scantron form will be counted and justifications can be minimal.

You have **two hours** to work on all **16** problems. Please do all your work in this exam booklet.

The Honor Principle requires that you neither give nor receive any aid on this exam.

FERPA Waiver: By my signature I relinquish my FERPA rights in the following context: My exam may be returned en masse with others present in the classroom. I acknowledge that I understand my score may be visible to others. If I choose not to relinquish my FERPA rights, I understand that I will have to present my student ID at my instructor's office to retrieve my exam.

Signature:______________________________

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Your name (please print): ______________________________

Problem	Points	Score
MC	60	
13	10	
14	10	
15	10	
16	10	
Total	**100**	

MULTIPLE CHOICE QUESTIONS

(1) The value of

$$\int_0^{\frac{4}{\pi}} \int_0^{\pi} 5(xy + \pi \sin(x))\, dx\, dy$$

is

(a): -60
(b): -15
(c): 15
(d): 60

(2) Find the volume of the solid E bounded by $z = 3 + x^2 + y^2$ and $z = 6$.

(a): $-27\frac{\pi}{2}$
(b): 9π
(c): $9\frac{\pi}{2}$
(d): $\frac{3\pi}{2}$

(3) Using polar coordinates, the integral

$$\iint_D (x + 4y^2)\, dA$$

where D is the top half of the circle of radius two centered at the origin, is equivalent to

(a):

$$\int_0^{2\pi} \int_0^2 r^2(\cos(\theta) + 4r\sin^2(\theta))\, dr\, d\theta$$

(b):

$$\int_0^{\pi} \int_0^2 r^2(\cos(\theta) + 4r\sin^2(\theta))\, dr\, d\theta$$

(c):

$$\int_0^{2\pi} \int_0^2 r(\cos(\theta) + 4r\sin^2(\theta))\, dr\, d\theta$$

(d):

$$\int_0^{\pi} \int_0^2 r(\cos(\theta) + 4r\sin^2(\theta))\, dr\, d\theta$$

(4) Consider the region D in the first quadrant of the xy-plane bounded between the lines $x = 0$, $y = 0$ and $y = -8x + 16$. Which of the following describes this region in polar coordinates.

(a): $\frac{\pi}{2} \leq \theta \leq \pi, 0 \leq r \leq \frac{16}{\sin(\theta)+8\cos(\theta)}$

(b): $0 \leq \theta \leq \frac{\pi}{2}, 0 \leq r \leq \frac{16}{\sin(\theta)+8\cos(\theta)}$

(c): $\frac{\pi}{2} \leq \theta \leq \pi, 0 \leq r \leq \frac{8}{\sin(\theta)+16\cos(\theta)}$

(d): $0 \leq \theta \leq \frac{\pi}{2}, 0 \leq r \leq \frac{8}{\sin(\theta)+16\cos(\theta)}$

(5) Let $\vec{\mathbf{v}} = (3, -1, 2)$ and $\vec{\mathbf{w}} = (1, 2, 5)$, then $\vec{\mathbf{v}} \times \vec{\mathbf{w}}$ is equal to

(a): $9\mathbf{i} + 13\mathbf{j} + 7\mathbf{k}$
(b): $9\mathbf{i} + 13\mathbf{j} - 7\mathbf{k}$
(c): $9\mathbf{i} - 13\mathbf{j} + 7\mathbf{k}$
(d): $-9\mathbf{i} - 13\mathbf{j} + 7\mathbf{k}$

(6) Evaluate

$$\iint_D dA$$

where D is the region under $y = 4 - 2x^2$ in the first quadrant.

(a): $-\frac{8}{3}\sqrt{2}$
(b): $\frac{8}{3}\sqrt{2}$
(c): 0
(d): $\frac{16}{3}\sqrt{2}$

(7) By switching the order of integration then

$$\int_0^3 \int_x^3 f(x,y)\ dy\ dx + \int_{-3}^0 \int_x^3 f(x,y)\ dy\ dx$$

is equivalent to

(a):

$$\int_0^3 \int_{-y}^y f(x,y)\ dx\ dy$$

(b):

$$\int_{-3}^3 \int_{-y}^y f(x,y)\ dx\ dy$$

(c):

$$\int_{-3}^3 \int_{-3}^y f(x,y)\ dx\ dy$$

(d):

$$\int_x^3 \int_0^3 f(x,y)\ dx\ dy + \int_x^3 \int_{-3}^0 f(x,y)\ dx\ dy$$

(8) The direction of greatest *increase* for $f(x,y) = x^3y + 12x^2 - 8y$ at $(1,-5)$ is

(a): $\frac{(9,-7)}{\sqrt{130}}$

(b): $-\frac{(9,-7)}{\sqrt{130}}$

(c): $\frac{(9,-7)}{130}$

(d): $-\frac{(9,-7)}{130}$

(9) The volume of the region bounded by the surface $z = 1 + y^2$ and the planes $x = 0$, $y = 0$, $z = 0$, and $x + y = 1$ is
(a): $13/12$
(b): $7/12$
(c): $\pi/12$
(d): $\sqrt{3}/12$

(10) The value of the double integral $\iint_R e^{2x^2+y^2}\ dA$, where $R = [0,1] \times [0,2]$, lies in the interval
(a): $[0,2]$
(b): $[2e^6, \infty]$
(c): $[2, 2e^6]$
(d): $[-e, 0]$

(11) The equation of the tangent plane to the surface $z = x^3y - 3y^2$ at the point $(1, 2, -10)$ is equal to

(a): $z - 10 = 5(x - 1) + 2(y - 2)$
(b): $z - 6x + 11y - 26 = 0$
(c): $z - 10 = 2(x - 1) + 4(y - 2)$
(d): $z - 3x + 2y - 9 = 0$

(12) A lamina occupies the part of the disk $x^2 + y^2 \leq 1$ in the first quadrant. If the density at any point is proportional to its distance from the origin, then the center of mass of the lamina is

(a):
(b):
(c):
(d):

NON-MULTIPLE CHOICE QUESTIONS

(13) Suppose a mountain is described by the function $z = 10x^2y - 5x^2 - 4y^2 - x^4 - 2y^4$ and that you are standing at the point $(1, 1, -2)$. The positive x-axis points east and the positive y-axis points north. If you walk in the northeast direction what angle above the horizontal does your path make?

(14) Evaluate the integral $\int_0^1 \int_y^1 e^{-3x^2}\, dxdy$.

(15) Consider the integral $\int_0^1 \int_0^{1-x^2} \int_0^{1-x} f(x, y, z)\, dydzdx$, where f is a continuous function. Rewrite this integral as an equivalent iterated integral with respect to $dzdxdy$.

(16) Find the volume bounded between the sphere of radius a centered at $(0, 0, 0)$ and the cone $z = \sqrt{x^2 + y^2}$.

(17) Find the average value of the function $f(x, y, z) = e^{x+y}$ over the tetrahedron in 3-space determined by the points $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$.