Math 13 Spring 2011

Multivariable Calculus

Final Exam

Saturday June 4, 11:30-2:30 PM

Your name (please print): ____

Instructor (circle one): Sutton, Yang

Instructions: This is a closed book, closed notes exam. **Use of calculators is not permitted**. You must justify all of your answers on free-response questions to receive credit, unless instructed otherwise in a given problem. On the multiple choice questions, only the answer you mark on the scantron form will be counted and justifications can be minimal.

You have **three hours** to work on all **16** problems. Please do all your work in this exam booklet.

The Honor Principle requires that you neither give nor receive any aid on this exam.

FERPA Waiver: By my signature I relinquish my FERPA rights in the following context: My exam may be returned en masse with others present in the classroom. I acknowledge that I understand my score may be visible to others. If I choose not to relinquish my FERPA rights, I understand that I will have to present my student ID at my instructor's office to retrieve my exam.

Signature:____

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Your name (please print): ______________________________

Problem	Points	Score
MC	40	
11	10	
12	10	
13	10	
14	10	
15	10	
16	10	
Total	**100**	

MULTIPLE CHOICE QUESTIONS

(1)
$$\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{\sqrt{x^2+y^2}}^{2} xz\ dzdxdy =$$

(a): 0
(b): π
(c): $-\pi$
(d): The integral cannot be evaluated.

(2) Does there exist a function $f(x,y,z)$ defined on all of $\mathbb{R}^3$ with
$$\nabla f(x,y,z) = \langle 2xe^{x^2}, z\sin(y^2), z^{1234}\rangle?$$

(a): Yes, such a function does exist.
(b): No, such a function does not exist because $\nabla \times \nabla f \neq \mathbf{0}$.
(c): No, such a function does not exist because $\nabla f \neq \mathbf{0}$.
(d): No, such a function does not exist because $\nabla \cdot \nabla f \neq 0$.

(3) Let $\mathbf{F} = \langle e^{yz}, \sin(xz^2), z^{1234} \rangle$. Does there exist a vector field $\mathbf{G}$ defined on all of $\mathbb{R}^3$ with $\mathbf{F} = \nabla \times \mathbf{G}$?

(a): No, because $\nabla \times \mathbf{F} \neq \mathbf{0}$.
(b): Yes, because $\nabla \cdot \mathbf{F} = 0$.
(c): No, because $\mathbf{F}(0,0,0) \neq \mathbf{0}$.
(d): No, because $\nabla \cdot \mathbf{F} \neq 0$.

(4) Let $f(x,y,z) = x + y^2 + z^3$. Then, at the point $(0,1,1)$ the function f *decreases* fastest in the direction of

(a): $\langle -1, -1, -1 \rangle$
(b): $\langle -1, -2, -3 \rangle$
(c): $\langle 1, 2, 3 \rangle$
(d): $\langle 1, 0, 1 \rangle$

(5) The arclength of the curve $\mathbf{r}(t) = \langle 7\sqrt{2}t, e^{-7t}, e^{7t} \rangle, 0 \le t \le 1$ is equal to:

(a): $e^7 - e^{-7}$
(b): $e - e^{-1}$
(c): 12
(d): π

(6) Let S be the part of the surface $z = 3x^2 + 3y^2$ below the plane $z = 27$ oriented with the *downward* pointing unit normal vector. If $\mathbf{F} = 2x^2\,\mathbf{i} + 3\cos(z)\,\mathbf{j} + e^{10(x+z)}\,\mathbf{k}$, then

$$\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S} =$$

(a): $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is parametrized by $\sigma(t) = (3\cos(-t), 3\sin(-t), 27)$, $0 \le t \le 2\pi$.
(b): $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is parametrized by $\sigma(t) = (3\cos(t), 3\sin(t), 27)$, $0 \le t \le 2\pi$.
(c): $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is parametrized by $\sigma(t) = (\cos(-t), \sin(-t), 0)$, $0 \le t \le 2\pi$.
(d): $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is parametrized by $\sigma(t) = (\cos^2(t), \sin^2(-t), 0)$, $0 \le t \le 2\pi$.

(7) Find the surface area of S, where S is the portion of the sphere of radius 3 centered at the origin which is inside the cylinder $x^2 + y^2 = 5$.

(a): 6π
(b): 12π
(c): 18π
(d): 24π

(8) Let C be the unit circle centered at the origin of the xy-plane oriented counter-clockwise and $\mathbf{F} = \left\langle -\frac{y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$. Then

$$\int_C \mathbf{F} \cdot \, d\mathbf{r} =$$

(a): 0
(b): -2π
(c): 2π
(d): none of the above

(9) Let C be the boundary of the rectangle defined by $-1 \leq x \leq 1, -1 \leq y \leq 2$, oriented *clockwise*. Then

$$\int_C x^3\, dx + (x + \sin y)\, dy =$$

(a): 6
(b): -6
(c): 2
(d): -2

(10) Which of the following is an equation of the plane tangent to the surface $x^2 + y^2 - 3z = 2$ at the point $(-2, -4, 6)$?

(a): $4x + 8y + 3z = 0$
(b): $-2x - 4y + 6z - 2 = 0$
(c): $4x + 8y + 3z + 22 = 0$
(d): $x + y + z = 0$

NON-MULTIPLE CHOICE QUESTIONS

(11) Let S be the surface $z = \sqrt{x^2 + y^2}$, $0 \leq x^2 + y^2 \leq 4$, with upwards pointing orientation. Let $\mathbf{F} = \langle y, -x, 1 \rangle$.

(a) What is the surface area of S?

(b) Let $\mathbf{n}$ be the unit normal vector giving the orientation of S described above. For every point (x, y, z) on S, calculate $\mathbf{F} \cdot \mathbf{n}$ as a function of x, y, z.

(c) Calculate

$$\iint_S \mathbf{F} \cdot d\mathbf{S}.$$

(12) Let $\mathbf{F} = \langle x^2 + \cos y, xz, e^{xy} \rangle$ and S be the surface of the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$ and $x+y+z = 2$, with outwards pointing orientation. Evaluate

$$\iint_S \mathbf{F} \cdot d\mathbf{S}.$$

(13) Find

$$\iiint_H (9 - x^2 - y^2)\, dV$$

where H is the solid hemisphere $x^2 + y^2 + z^2 \leq 9, z \geq 0$.

(14) Let S be the surface defined by the equation $z = \dfrac{1}{xy}$ lying over the part of the xy-plane satisfying the inequalities

$$\frac{(x-3)^2}{4} + \frac{(y-4)^2}{4} \leq 1,$$

with upwards pointing orientation. Let $\mathbf{F} = \left\langle \dfrac{x}{z}, \dfrac{y}{z}, xyz \right\rangle$. Calculate

$$\iint_S \mathbf{F} \cdot d\mathbf{S}.$$

(15) Let S be the surface of the paraboloid $z = x^2 + y^2$ for $0 \leq z \leq 3$ oriented by the *upward* pointing normal, and consider the continuously differentiable vector field

$$\mathbf{F} = \langle \cos(y+z) - 2y, -x\sin(y+z) + y, \cos(x+y) \rangle.$$

One can compute that

$$\operatorname{curl} \mathbf{F} = \langle -\sin(x+y) + x\cos(y+z),\ \sin(x+y) - \sin(y+z),\ 2 \rangle.$$

Evaluate

$$\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}.$$

(16) Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x; for instance, $\lfloor 2.5 \rfloor = 2$, $\lfloor \pi \rfloor = 3$, $\lfloor 1 \rfloor = 1$, $\lfloor -.2 \rfloor = -1$. Let D be the disc $x^2 + y^2 \leq 9$. Compute

$$\iint_D \lfloor \sqrt{x^2 + y^2} \rfloor \, dA.$$

(Hint: What are the level sets of the integrand?)