WRITTEN HOMEWORK #6, DUE 2/20/2012 AT 4PM

You may turn this assignment at the homework boxes on the bottom floor of Kemeny or at the beginning of class on Monday. Please staple your assignment before turning it in. Remember that you need to provide correct and reasonably complete details to receive full credit. The problems are taken from the 7th edition of Stewart's *Calculus*, although occasionally a problem will be modified to be slightly different from its textbook counterpart.

(1) (Chapter 16.3, #29) Show that if the vector field $\mathbf{F} = \langle P, Q, R \rangle$ is conservative and P, Q, R have continuous first-order partial derivatives, then

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}, \frac{\partial P}{\partial z} = \frac{\partial R}{\partial x}, \frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y}.$$

(2) (Chapter 16.3, #14) Find a potential function $f(x, y)$ for $\mathbf{F} = \langle (1+xy)e^{xy}, x^2 e^{xy} \rangle$, and evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is given by $\mathbf{r}(t) = \langle \cos t, 2 \sin t \rangle, 0 \leq t \leq \pi/2$.

(3) (Chapter 16.3, #36a) Suppose that $\mathbf{F}$ is an inverse square field; that is,

$$\mathbf{F}(\mathbf{r}) = \frac{c\mathbf{r}}{|\mathbf{r}|^3}$$

for some constant c, where $\mathbf{r} = \langle x, y, z \rangle$. Find the work done by $\mathbf{F}$ in moving an object from point P_1 along a path to a point P_2 in terms of the distances d_1, d_2 from these points to the origin.

(4) (Chapter 16.4, #2) Evaluate the line integral below by using two methods: direct evaluation, and Green's Theorem, and check that the answers are identical.

$$\int_C xy\, dx + x^3\, dy,$$

where C is the rectangle (with positive orientation) with vertices $(0, 0), (3, 0), (3, 1)(0, 1)$.

(5) (Chapter 16.4, #22) Let D be a region bounded by a simple closed path C in the xy-plane. Use Green's Theorem to prove that the coordinates $(\overline{x}, \overline{y})$ of the centroid (the centroid is the center of mass of D, if we assume that D is a lamina of uniform density) of D are

$$\overline{x} = \frac{1}{2A} \int_C x^2\, dy, \overline{y} = -\frac{1}{2A} \int_C y^2\, dx.$$