# WRITTEN HOMEWORK #5, DUE 2/13/2012 AT 4PM

You may turn this assignment at the homework boxes on the bottom floor of Kemeny or at the beginning of class on Monday. (Notice the change in date due to Winter Carnival! However, HW 6 will be due on Friday, as usual.) Please staple your assignment before turning it in. Remember that you need to provide correct and reasonably complete details to receive full credit. The problems are taken from the 7th edition of Stewart's *Calculus*, although occasionally a problem will be modified to be slightly different from its textbook counterpart.

(1) (Chapter 15.10, Problem#14) Let $R$ be the region bounded by hyperbolas $y = 1/x, y = 4/x$, and the lines $y = x, y = 4x$, in the first quadrant. Find equations for a transformation $T$ that maps a rectangular region $S$ in the $uv$-plane onto $R$, where the sides of $S$ are parallel to the $u, v$ axes. In addition, sketch $R$ and $S$.

(2) (Chapter 15.10, Problem#18) Evaluate $\iint_R (x^2 - xy + y^2)\, dA$, where $R$ is the region bounded by the ellipse $x^2 - xy + y^2 = 2$. Use the change of variables $x = \sqrt{2}u - \sqrt{2/3}v, y = \sqrt{2}u + \sqrt{2/3}v$.

(3) (Chapter 16.2, Problem #16) Evaluate the line integral $\int_C (y + z)\, dx + (x + z)\, dy + (x + y)\, dz$, where $C$ is the concatenation of the line segment from $(0, 0, 0)$ to $(1, 0, 1)$ with the line segment from $(1, 0, 1)$ to $(0, 1, 2)$.

(4) (Chapter 16.2, #42) The force exerted by a unit electric charge at the origin on a charged particle at a point $(x, y, z)$ is $\mathbf{F}(\mathbf{r}) = \mathbf{r}/|\mathbf{r}|^3$, where $\mathbf{r} = \langle x, y, z\rangle$. (The textbook multiplies this equation by a constant $K$, but in this problem just assume $K = 1$.) Find the work done as the particle moves from a straight line from $(2, 0, 0)$ to $(2, 1, 5)$.

(5) (Chapter 16.2, #52) Experiments show that a steady current $I$ in a long wire produces a magnetic field $\mathbf{B}$ that is tangent to any circle that lies in the plane perpendicular to the wire and whose center is the axis of the wire (consult the book for a picture). *Ampére's Law* relates the current to its magnetic effects and states that

$$\int_C \mathbf{B} \cdot d\mathbf{r} = \mu_0 I,$$

where $I$ is the net current that passes through any surface bounded by a closed curve $C$, and $\mu_0$ is a constant called the permeability of free space. By taking $C$ to be a circle with radius $r$, show that the magnitude $B = |\mathbf{B}|$ of the magnetic field at a distance $r$ from the center of the wire is

$$B = \frac{\mu_0 I}{2\pi r}.$$

(6) (Chapter 16.3, #29) (Note: This question is not required for this week's problem set, and will be moved to next week's problem set.) Show that if the vector field $\mathbf{F} = \langle P, Q, R\rangle$ is conservative and $P, Q, R$ have continuous first-order partial derivatives, then

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}, \frac{\partial P}{\partial z} = \frac{\partial R}{\partial x}, \frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y}.$$