MATH 13, WINTER 2011
WRITTEN HOMEWORK #6

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This assignment will be due on **Friday, February 18** at 12:30 p.m. in the boxes outside 105 Kemeny. Look for the column of boxes labeled "Math 13, Winter 2011" and put your assignment in the left ("IN") column corresponding to the first letter of your family name (A-F, G-M, N-S, T-Z).

Remember to show your work. A correct answer with no work shown will receive minimal credit. Your solutions should be detailed enough that any of your classmates could understand them simply by reading them.

(1) (17.2, #32, part (a)) Find the work done by the force field $\mathbf{F}(x, y) = x^2\mathbf{i} + xy\mathbf{j}$ on a particle that moves once around the circle $x^2 + y^2 = 4$ oriented in the counterclockwise direction.

(2) (17.3, #26) Let $\mathbf{F} = \nabla f$, where $f(x, y) = \sin(x - 2y)$. Find curves C_1 and C_2 that are not closed and satisfy the equations $\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = 0$ and $\int_{C_2} \mathbf{F} \cdot d\mathbf{r} = 1$.

(3) (17.3, #34)

(a) Suppose that $\mathbf{F}$ is an inverse square force field, that is,

$$\mathbf{F}(\mathbf{r}) = \frac{c\mathbf{r}}{|\mathbf{r}|^3}$$

for some constant c, where $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Find the work done by $\mathbf{F}$ in moving an object from a point P_1 along a path to a point P_2 in terms of the distances d_1 and d_2 from these points to the origin.

(b) An example of an inverse square field is the gravitational field $\mathbf{F} = -\frac{mMG\mathbf{r}}{|\mathbf{r}|^3}$ discussed in Example 4 in Section 17.1. Use part (a) to find the work done by the gravitational field when the earth moves from aphelion (at a maximum distance of 1.52×10^8 km from the sun) to perihelion (at a minimum distance of 1.47×10^8 km). Use the values $m = 5.97 \times 10^{24}$ kg, $M = 1.99 \times 10^{30}$ kg, and $G = 6.67 \times 10^{-11}\ \frac{N \cdot m^2}{kg^2}$.

(c) Another example of an inverse square field is the electric force field $\mathbf{F} = \frac{\epsilon qQ\mathbf{r}}{|\mathbf{r}|^3}$ discussed in Example 5 in Section 17.1. Suppose that an electron with a charge of -1.6×10^{-19} C is located at the origin. A positive unit charge is positioned a distance 10^{-12} m from the electron and moves to a position half that distance from the electron. Use part (a) to find the work done by the electric force field. Use the value $\epsilon = 8.985 \times 10^9$.

(4) Verify Green's Theorem for $P(x, y) = x$ and $Q(x, y) = xy$ where D is the unit disk $x^2 + y^2 \leq 1$.

Suggested problems: 17.1: 29-32; 17.2: 5, 7, 19, 37, 39; 17.3: 1, 3, 13, 19, 23-24; 17.4: 3, 11, 13