Paths, Curves and Arc Length

January 17, 2006

Definition of a path

Let $I = [a, b]$ be a closed interval for some numbers $a < b$. $I \subseteq \mathbb{R}$.

Definition: A **path in** $\mathbb{R}^n$ is a continuous function $\mathbf{x} : I \longrightarrow \mathbb{R}^n$ where $\mathbf{x}(a)$ and $\mathbf{x}(b)$ are the **endpoints** of the path $\mathbf{x}$.

Velocity, speed and acceleration

Let $\mathbf{x} : I \to \mathbb{R}^n$ be a differentiable path. Then

- The **velocity** $\mathbf{v}(t) = \mathbf{x}'(t)$.

- The **speed** is $\|\mathbf{v}(t)\|$.

- The **acceleration** is $\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{x}''(t)$.

Parametric equation of the tangent line

Let $\mathbf{x} : I \to \mathbb{R}^n$ be a path and $\mathbf{v}(t_0) \neq \mathbf{0}$. Then the parametric equation of the tangent line at t_0 to the path $\mathbf{x}$ is

$$\mathbf{l}(t) = \mathbf{x}(t_0) + (t - t_0)\mathbf{v}_0.$$

Length of a path

Definition: The **length** $L(\mathbf{x})$ of a differentiable path $\mathbf{x} : [a, b] \longrightarrow \mathbb{R}^n$ is the integral of its speed

$$L(\mathbf{x}) = \int_a^b \|\mathbf{x}'(t)\| \; dt$$