Double Integrals over General Regions

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Elementary Regions

Type 1: $D = \{(x, y) \mid , \gamma(x) \leq y \leq \delta(x), a \leq x \leq b\}$, where γ and δ are continuous function of x.

Type 2: $D = \{(x, y) \mid \alpha(y) \leq x \leq \beta(y), c \leq y \leq d\}$, where α and β are continuous functions of y.

Type 3: D can be written as both of type 1 or type 2.

Integrals over elementary regions

Let D be an elementary region in $\mathbb{R}^2$ and f continuous on D.

- If D is of type 1, then
$$\iint_D f\,dA = \int_a^b \int_{\gamma(x)}^{\delta(x)} f(x,y)\,dy\,dx.$$

- If D is of type 2, then
$$\iint_D f\,dA = \int_c^d \int_{\alpha(y)}^{\beta(x)} f(x,y)\,dx\,dy.$$