1. A lumberjack cuts a wedge out of a cylindrical tree of radius 2 feet by making two saw cuts to the tree's center, one horizontally and one at an angle of $\theta = \pi/6$ above the horizontal. Compute the volume of the wedge removed. Hint: Vertical slices are triangles.

2. Consider the solid that lies above the square $R = [0, 2] \times [0, 2]$ in the $xy$-plane, and below the elliptic paraboloid $z = 36 - x^2 - 3y^2$.
   (a) Estimate the volume of the solid by dividing $R$ into four equal squares with the lower left-hand corners of the squares as the sample points.
   (b) Estimate the volume by dividing $R$ into four equal squares with the upper right-hand corners as sample points.
   (c) What is the average of the answers from parts (a) and (b)?
   (d) Using iterated integrals, compute the exact volume of the solid.

3. Find the volume of the region inside the sphere $x^2 + y^2 + z^2 = 25$ and above the plane $z = 3$.

4. Redo problem 16 on page 319 by changing to the variables $u = xy$ and $v = \frac{y}{x}$.

5. Find the centroid of the portion of the ball $\rho \leq 1$ in the first octant.

6. Evaluate $\oint_C (-x^2y + \cos x^2)\, dx + xy^2\, dy$, where $C$ is the boundary of the region enclosed by the circles $x^2+y^2 = 4$ and $x^2+y^2 = 16$. The outer circle of the boundary is oriented counterclockwise, the inner clockwise.

7. Calculate the outward flux of the vector field $\mathbf{F}(x, y) = (x, y^2)$ across the square $S$ bounded by the lines $x = \pm 1$ and $y = \pm 1$. That is, compute $\oint_{\partial S} \mathbf{F} \cdot \mathbf{n}\, ds$, where $\mathbf{n}$ is the outward pointing unit normal and $\partial S$ is suitably oriented.

8. Find the counterclockwise circulation of the vector field $\mathbf{F} = (xy, y^2)$ around and the outward flux of $\mathbf{F}$ across the boundary of the region enclosed by the parabola $y = x^2$ and the line $y = x$. That is, compute the line integral of $\mathbf{F}$ around the boundary (oriented counterclockwise) of the region as well as the outward flux of $\mathbf{F}$ across the boundary.

9. Evaluate $\int_C (2xy^3 - y^2 \cos x)\, dx + (1 - 2y \sin x + 3x^2y^2)\, dy$, where $C$ is the portion of the parabola $x = \frac{\pi}{2} y^2$ from $(0, 0)$ to $(\frac{\pi}{2}, 1)$.

10. Let $y = f(x)$ be a function which is defined on the interval $[a, b]$ and satisfies $f(x) \geq 0$ for all $x$ in $[a, b]$. Parametrize the surface obtained by rotating the graph of $f$ about the $x$-axis. Write the integral which gives the area of this surface in as simple a form as possible.

11. Let $S$ be a surface in $\mathbf{R}^3$ that is actually a subset $D$ of the $xy$-plane. Show that the integral of a scalar function $f$ over $S$ reduces to the double integral of $f$ over $D$. What does the surface integral of a vector field $\mathbf{F}(x, y, z) = (f(x, y, z), g(x, y, z), h(x, y, z))$ over $S$ become?