Math 13 - First Hour Exam - January 30, 2002

Part I: Multiple choice. Each problem is worth 5 points.

1. The following is the tangent line to $\mathbf{c}(t) = (e^t, \sin t, \cos t)$ at $t_0 = 0$:

   (a) $(1, 1, 0)$

   (b) $(1 + t, 0, 1)$

   (c) $(1 + t, t, 1)$

   (d) $(t, 1, t)$

2. The following vector is normal to the plane $3(x - 1) + 2y - z = 4$

   (a) $(4, 0, 0)$

   (b) $(3, 0, 0)$

   (c) $(3, 2, -1)$

   (d) $(4, 0, -4)$

3. Consider the matrices: $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 3 \\ 1 & -1 & 5 \end{bmatrix}$,
   $C = \begin{bmatrix} 4 & 2 \\ 3 & -1 \\ 8 & 0 \end{bmatrix}$.

   Of the following matrix products, which make sense? AC, AB, CB, and BC.

   (a) CB and AB

   (b) AC, AB, and CB

   (c) all of them

   (d) AB, CB, and BC

4. Which of the following is NOT a gradient field?
   (a) $\mathbf{F} = (yz - 2y, xz - 2x, xy)$
   (b) $\mathbf{F} = (z^2 + y, x, zyx)$
   (c) Neither are gradient fields
   (d) Both are gradient fields

5. The path in the xy-plane of a particle following the ellipse $2x^2 + y^2 = 2$ in the counterclockwise direction is described by:
   (a) $\mathbf{c}(t) = (2\cos t, \sin t)$
   (b) $\mathbf{c}(t) = (\cos t, \sqrt{2}\sin t)$
   (c) $\mathbf{c}(t) = (\sqrt{2}\sin t, \cos t)$
   (d) $\mathbf{c}(t) = (2\sin t, \cos t)$

6. Let the acceleration of a particle in the plane be given by $\mathbf{a} = (24t, e^t)$. Suppose that it's initial velocity at $t = 0$ is $(1, 1)$, and it's initial position at $t = 0$ is $(2, 2)$. Then the particle is moving on the following path:
   (a) $(12t^2, e^t)$
   (b) $(4t^3 + t + 2, e^t + 1)$
   (c) $(4t^3 + t + 2, e^t + t + 2)$
   (d) $(4t^3, e^t)$

7. True False: State whether the following statements are true or false, in the order (1), (2), (3).

   (1) A flow line of a vector field is a curve which the field is perpendicular to at each point of the curve.

   (2) If $\mathbf{a}$ and $\mathbf{b}$ are perpendicular, then $\mathbf{a} \cdot \mathbf{b} = 0$

   (3) A plane is perpendicular to the cross product of any two vectors in it.

   (a) TTT
   (b) TTF
   (c) FTT
   (d) FTF

8. Let $\mathbf{F}$ and $\mathbf{G}$ be vector fields, and let f be a scalar function of three variables ($\mathbf{F}$ and $\mathbf{G} : \Re^3 \longrightarrow \Re^3$ and $f : \Re^3 \longrightarrow \Re$). Do the following statements make mathematical sense, ie, can the operations be performed? Answer Y or N in the order (1) - (5).

   (1) $div(\mathbf{F} \times \mathbf{G})$

   (2) $\nabla f \times \mathbf{F}$

   (3) the curl of $\mathbf{F} \cdot \mathbf{G}$

   (4) the cross product of a vector field and its curl

   (5) the dot product of ∇f and $div(\mathbf{F})$

   (a) YNNYY

   (b) YYNYN

   (c) NYNYN

   (d) YYYNY

9. Which of the following level surfaces is expressible as a graph $z = f(x, y)$ about the point $(0, 1, 1)$?

   (a) $xze^y + \frac{1}{3}z^3 - zy = 0$

   (b) $\frac{1}{4}z^4 y + zcos(x^2) = 0$

   (c) Both of the above are expressible as $z = f(x, y)$

   (d) Neither of the above are expressible as $z = f(x, y)$

10. Match the equations to the surfaces (or parts of surfaces) that they map in $\Re^3$.

(i) $z = x^2 + y^2$	(α) cone
(ii) $z = \sqrt{x^2 + y^2}$	(β) plane
(iii) $3 = x^2 + y^2$	(γ) cylinder
(iv) $z = \sqrt{4 - x^2 - y^2}$	(δ) sphere
(v) $z = 5 - x + 2y$	(ϵ) paraboloid

   Which of the following is true?

   (a) (i) - γ, (ii) - α, (iii) - ϵ, (iv) - δ, (v) - β

   (b) (i) - ϵ, (ii) - δ, (iii) - γ, (iv) - β, (v) - α

   (c) (i) - α, (ii) - ϵ, (iii) - δ, (iv) - β, (v) - γ

   (d) (i) - ϵ, (ii) - α, (iii) - γ, (iv) - δ, (v) - β

Part II: You can earn partial credit on the next five problems.

11. (10 points) Location on a particular mountain is given by points in the x-y plane where north is in the positive y direction. The elevation in feet above sea level at a point (x, y) is given by $g(x, y) = 10000 - 2x^2 - y^2$. If you are standing at point $(1, 1)$,

(a) What is the rate of change of elevation in the south-eastern direction (ie, in direction of vector $\mathbf{i} - \mathbf{j}$)?

(b) In what direction is the mountain decreasing in elevation the fastest from point $(1, 1)$?

12. (10 points) Suppose that $f(x, y, z) = (2xy, e^{xz})$ and $g(u, v) = (\cos u, vu)$.

(a) If $f : \Re^n \longrightarrow \Re^m$ and $g : \Re^p \longrightarrow \Re^q$, what are n, m, p and q?

(b) Which of the compositions, $f \circ g$ or $g \circ f$, is (are) defined?

(c) For any compositions that are defined, compute their derivative matrix.

13. (10 points) Find the arc length of $\mathbf{c}(t) = (1, 3t^2, t^3)$ from $(1, 0, 0)$ to $(1, 12, 8)$.

14. (10 points) Find the equation of the tangent plane to the surface $z = e^x(\sin y + 1)$ at $(0, \frac{\pi}{2}, 2)$.

15. (10 points) Find the divergence and curl of

$$\mathbf{F}(x, y, z) = (x \sin z, -2xz, z^2 + 2y)$$

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