Part I: Multiple choice. Each problem is worth 5 points.

1. The integral $\int\int_R (x^2 + e^y)\ dx\ dy$, where $R = [0,1] \times [0, \ln 2]$ is equal to

(a) $\frac{1}{3}\ln 2$

(b) $\ln 2 + 1$

(c) $\frac{1}{3}\ln 2 + 1$

(d) $\ln 2$

2. The integral $\int_{-1}^{4}\int_{y+1}^{5} e^{x^2}\ dx\ dy$ is equal to

(a) cannot be evaluated since $e^{x^2}$ does not have antiderivatives in terms of elementary functions.

(b) $5e^{25} - e^5 + 1$

(c) $e^{25} - e^{16} + e^{-1}$

(d) $\frac{1}{2}e^{25} - \frac{1}{2}$

3. Consider the integral $\int_0^2\int_0^{4-2y} f(x,y)\ dx\ dy$. Changing limits of integration makes it equal to

(a) $\int_0^4\int_{\frac{4-x}{2}}^{2} f(x,y)\ dy\ dx$

(b) $\int_0^{\frac{4-x}{2}}\int_0^4 f(x,y)\ dy\ dx$

(c) $\int_0^4\int_0^{\frac{4-x}{2}} f(x,y)\ dy\ dx$

(d) $\int_0^{4-2y}\int_0^2 f(x,y)\ dy\ dx$

4. Consider the triple integral $\int\int\int_W f(x,y,z)\ dx\ dy\ dz$ over a region $W$ in space. When writing it as an iterated integral $\int\int\int f(x,y,z)dx\ dz\ dy$ the limits of integration are of the form ($a$ and $b$ are real numbers)

(a) $h_1(x,z) \leq y \leq h_2(x,z)$, $g_1(y) \leq z \leq g_2(y)$, $a \leq x \leq b$

(b) $h_1(y,z) \leq x \leq h_2(y,z)$, $g_1(y) \leq z \leq g_2(y)$, $a \leq y \leq b$

(c) $h_1(y,z) \leq x \leq h_2(y,z)$, $g_1(z) \leq y \leq g_2(z)$, $a \leq z \leq b$

(d) $h_1(x,y) \leq z \leq h_2(x,y)$, $g_1(y) \leq x \leq g_2(y)$, $a \leq y \leq b$

5. Given the point $(\sqrt{2}, 0, 1)$ in Cartesian coordinates, its cylindrical coordinates are

(a) $(\sqrt{2}, 0, 1)$

(b) $(\sqrt{2}, \pi, 1)$

(c) $(\sqrt{3}, 0, 1)$

(d) $(\sqrt{2}, \frac{\pi}{2}, 1)$

6. Given the point $(-1, 0, 1)$ in Cartesian coordinates, its spherical coordinates are

(a) $(1, 0, \frac{\pi}{4})$

(b) $(\sqrt{2}, 0, \frac{\pi}{4})$

(c) $(\sqrt{2}, \frac{\pi}{4}, 0)$

(d) $(\sqrt{2}, \pi, \frac{\pi}{4})$

7. Let $D$ be the region bounded by $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. The value of the integral $\int\int_D (x^2 + y^2 + 1)\, dx\, dy$ is

(a) $\frac{20}{3}\pi$

(b) $\frac{15}{2}\pi$

(c) $\frac{21}{2}\pi$

(d) $3\pi$

8. Match the integrals with the type of coordinates which make them easiest to do.

(1) $\int\int\int_E (x + y + z)\, dx\, dy\, dz$, where $E = [0, 1] \times [0, 1] \times [0, 1]$

(2) $\int\int_D e^{\sqrt{x^2+y^2}}\, dx\, dy$, where $D$ is: $x^2 + y^2 \leq 1$

(3) $\int\int\int_E (x^2 + y^2 + z^2)\, dx\, dy\, dz$, where $E$ is: $x^2 + y^2 \leq 1,\ 0 \leq z \leq 1$

(4) $\int\int\int_E (x^2 + y^2 + z^2)\, dx\, dy\, dz$, where $E$ is: $x^2 + y^2 + z^2 \leq 1$

(i) polar coordinates

(ii) spherical coordinates

(iii) cartesian coordinates

(iv) cylindrical coordinates

(a) 1 – iii, 2 – i, 3 – ii, 4 – ii

(b) 1 – iv, 2 – i, 3 – iv, 4 – ii

(c) 1 – iii, 2 – iii, 3 –iv, 4 – ii

(d) 1 - iii, 2 –i, 3 – iv, 4 – ii

9. The line integral of $F = \nabla f$, where $f(x, y, z) = x^2 + y^2 + z^2$, along the path $c(t) = (e^t \cos t, e^t \sin t, 3)$, $0 \leq t \leq \frac{\pi}{2}$, is equal to

(a) $e^{\pi} - 1$

(b) 0

(c) 1

(d) $e^{\frac{\pi}{2}} - 1$

10. Which of the following is false?

(a) The value of the line integral of a vector field along a path between two given points depends on the path chosen.

(b) $\int_{c_1} F \cdot ds = \int_{c_2} F \cdot ds$ whenever $c_1$ and $c_2$ are reparametrizations of the same curve.

(c) $\int_{c_1} F \cdot ds = \int_{c_2} F \cdot ds$ for all curves $c_1$ and $c_2$ with common start points and common end points.

(d) If $F$ is a gradient field and $(x_0, y_0, z_0)$ and $(x_1, y_1, z_1)$ are fixed points in space, then the line integral of $F$ is the same along all curves starting at $(x_0, y_0, z_0)$ and ending at $(x_1, y_1, z_1)$.

Part II: You can earn partial credit on the next five problems.

11. (9 points) Evaluate

$$\iiint_E z^2 \, dx \, dy \, dz,$$

where $E$ is the region inside the cylinder $x^2 + y^2 \leq 1$ bounded below by the plane $z = 0$ and above by the paraboloid $z = x^2 + y^2$.

12. (17 points) Calculate the volume of the solid inside $x^2 + y^2 + z^2 = 1$ and outside $z^2 = x^2 + y^2$.

13. (10 points) Rewrite the integral

$$\int_0^2 \int_0^1 \int_0^{2-2x} f(x, y, z) \, dy \, dx \, dz$$

as

$$\int_a^b \int_{g_1(z)}^{g_2(z)} \int_{h_1(y,z)}^{h_2(y,z)} f(x, y, z) \, dx \, dy \, dz$$

14. (14 points) Find the work done by the force $F(x, y) = (x^2 - y^2)\,\mathbf{i} + 2xy\,\mathbf{j}$ in moving a particle counterclockwise around the square in the plane having corners $(0, 0)$, $(5, 0)$, $(5, 5)$ and $(0, 5)$.

The following problem is optional and the points are additional credit.

15. (10 points) (i) Let $F = x^2\,\mathbf{i} - xy\,\mathbf{j} + \mathbf{k}$. Evaluate the line integral of $F$ along

(a) $c_1(t) = (1 + 2t)\,\mathbf{i} + \mathbf{k}$, $-1 \leq t \leq 1$

(b) $c_2(t) = \cos t\,\mathbf{i} + (1 + \sin t)\,\mathbf{k}$, $-\pi \leq t \leq 0$

(ii) Does there exist a function $f : \mathrm{R}^3 \to \mathrm{R}$ such that $\frac{\partial f}{\partial x} = x^2$, $\frac{\partial f}{\partial y} = -xy$ and $\frac{\partial f}{\partial z} = 1$? Explain.