Math 13 Homework #6
Due Wednesday, May 8th

(1) Compute the scalar line integrals $\int_C f(x,y,z)ds$ for the following functions f and curves C.

 (a) $f(x,y,z) = 3x - 2y + z$; C is given by $\mathbf{r}(t) = (2+t, 2-t, 2t)$ for $0 \leq t \leq 1$.
 (b) $f(x,y,z) = xe^{z^2}$; C is the triangle with vertices $(0,0,1)$, $(0,2,0)$ and $(1,1,1)$ traversed in that order, starting at $(0,0,1)$.
 (c) $f(x,y,z) = x^2 + y^2 + 3z$ where C is the unit circle in the plane $z = 1$, traversed counterclockwise.

(2) Evaluate the following line integrals.

 (a) $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = \langle e^{y-x}, e^{2x} \rangle$ and C is the line segment going from $(0,0)$ to $(2,2)$.
 (b) $\int_C z dx + x^2 dy + y dz$, $\mathbf{r}(t) = (\cos(t), \tan(t), t)$ for $0 \leq t \leq \frac{\pi}{4}$.

(3) Evaluate

$$\int_C 2xyz dx + x^2 z dy + x^2 y dz,$$

where C is parameterized by $\mathbf{r}(t) = (t,t,0)$, $0 \leq t \leq 1$.

How would your answer change if $\mathbf{r}(t)$ was replaced by $\mathbf{r}(t) = (1-t, 1-t, t^2 - t)$?

(4) Let $\mathbf{F} = \left\langle \frac{x}{x^2+y^2}, \frac{y}{x^2+y^2} \right\rangle$, and let $\mathcal{D} = \{(x,y) : (x,y) \neq (0,0)\}$.

 (a) Is $\mathcal{D}$ simply connected?
 (b) Show that $\mathbf{F}$ satisfies the cross-partial condition; i.e. $\frac{\partial F_1}{\partial y} = \frac{\partial F_2}{\partial x}$.
 (c) Show that $\mathbf{F}$ is conservative on $\mathcal{D}$ by finding a potential function.
 (d) Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is any smooth curve from $(1,0)$ to $(2,2)$.