Math 13 Homework #5
Due Wednesday, May 1st

(1) Find a mapping G that maps the disk $u^2 + v^2 \leq 1$ onto the interior of the ellipse $(x/a)^2 + (y/b)^2 \leq 1$. Then use the Change of Variables formula to show that the area of the ellipse is πab.

(2) Let F be the map from the xy-plane to the uv-plane defined by

$$F(x, y) = (x + y, y - 2x),$$

and let G be its inverse.

(a) Compute $\text{Jac}(G)$.

(b) Sketch the domain $\mathcal{D} = \{(x, y) : 1 \leq x + y \leq 4, -4 \leq y - 2x \leq 1\}$.

(c) Compute

$$\iint_{\mathcal{D}} e^{x+y} \; dxdy$$

using the Change of Variables Formula with the map G.

(3) Calculate $\text{div}(\mathbf{F})$ and $\text{curl}(\mathbf{F})$ for the following functions F.

(a) $\mathbf{F} = \langle xy, yz, y^2 - x^3 \rangle$

(b) $\mathbf{F} = \sin(x + z)\mathbf{i} - ye^{xz}\mathbf{k}$

(4) Show the following identities, supposing that all necessary partial derivatives exist and are continuous:

(a) $\text{div}\,\text{curl}(\mathbf{F}) = 0$,

(b) $\text{div}(\mathbf{F} \times \mathbf{G}) = G \cdot \text{curl}(\mathbf{F}) - \mathbf{F} \cdot \text{curl}(\mathbf{G})$.

(5) Find a potential function for the following vector fields $\mathbf{F}$ or show that $\mathbf{F}$ is not conservative.

(a) $\mathbf{F} = \langle x, y \rangle$,

(b) $\mathbf{F} = \langle y, x \rangle$,

(c) $\mathbf{F} = \langle yz, xz, y \rangle$.