Math 13 Homework #3
Due Wednesday, April 17th

(1) Let $\mathcal{W}$ be the region bounded below by the plane $z = 1$ and above by the sphere $x^2 + y^2 + z^2 = 9$.

   (a) Use cylindrical coordinates to find the volume of $\mathcal{W}$.

   (b) Use spherical coordinates to find the volume of $\mathcal{W}$.

(2) Let $\mathcal{W}$ be a sphere S of radius a from which a central cylinder of radius b has been removed, where $0 < b < a$.

   (a) Sketch the region $\mathcal{W}$.

   (b) Calculate the height and volume of the cylinder.

   (c) Calculate the volume of just the sphere S as a triple integral using spherical coordinates.

   (d) Compute the volume of $\mathcal{W}$.

(3) Let $\mathcal{W}$ be the region within the cylinder $x^2 + y^2 = 2$ between $z = 0$ and the cone $z = \sqrt{x^2 + y^2}$. Calculate the integral of $f(x, y, z) = x^2 + y^2$ over $\mathcal{W}$, using spherical coordinates.

(4) Calculate the center of mass of the region bounded by $y^2 = -x + 4$ and $x = 0$, and mass density $\delta(x, y) = x$.

(5) Calculate the moment of inertia I_z of the box $\mathcal{W} = [-a, a] \times [-a, a] \times [0, H]$ assuming $\mathcal{W}$ has total mass M in kg, and uniform mass density δ in kg / m^3.