Math 13 Homework #2
Due Wednesday, April 10th

(1) Evaluate the integral

$$\iiint_{\mathcal{B}} \frac{x}{(y+z)^2}\, dV$$

where $\mathcal{B} = [0,2] \times [2,4] \times [-1,1]$.

(2) Let S be the solid in $\mathbb{R}^3$ bounded by $y = x^2$, $x = y^2$ and $z = x + y + 5$ and $z = 0$.

(a) Sketch the projections onto the $xy-$ and yz-planes.

(b) Compute the volume of S.

(3) At a given time, the temperature at any point in a cave (viewed in $\mathbb{R}^3$) is given by the equation $T(x,y,z) = \frac{xy+z}{10}$ in centigrade. Compute the average temperature in the section of the cave in the first octant, bounded by $z = 9 - x^2$ and $x = y$.

(4) Sketch the domain of integration for the integral

$$\int_0^3 \int_0^{\sqrt{9-y^2}} \sqrt{x^2+y^2}\ \ dxdy,$$

and compute the integral by changing to polar coordinates.

(5) Sketch the domain of integration for the integral

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{1}{\sqrt{x^2+y^2}}\ \ dydx,$$

and compute the integral by changing to polar coordinates.

(6) Let $\mathcal{W}$ be the region in $\mathbb{R}^3$ given by $x^2 + y^2 \leq 1$, $x \geq 0$, $0 \leq z \leq 2$. For the function $f(x,y,z) = xz$,

(a) Sketch the domain of integration.

(b) Set up the integral

$$\iiint_{\mathcal{W}} f(x,y,z)dV$$

in terms of x, y, and z.

(c) Use cylindrical coordinates to evaluate the integral.