Stokes' Theorem

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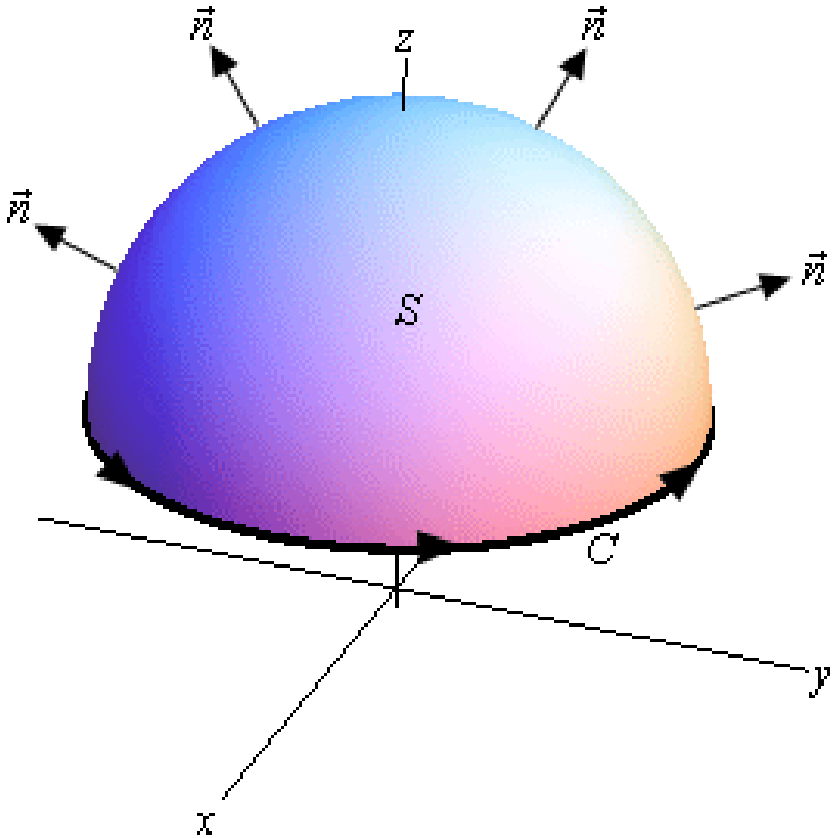
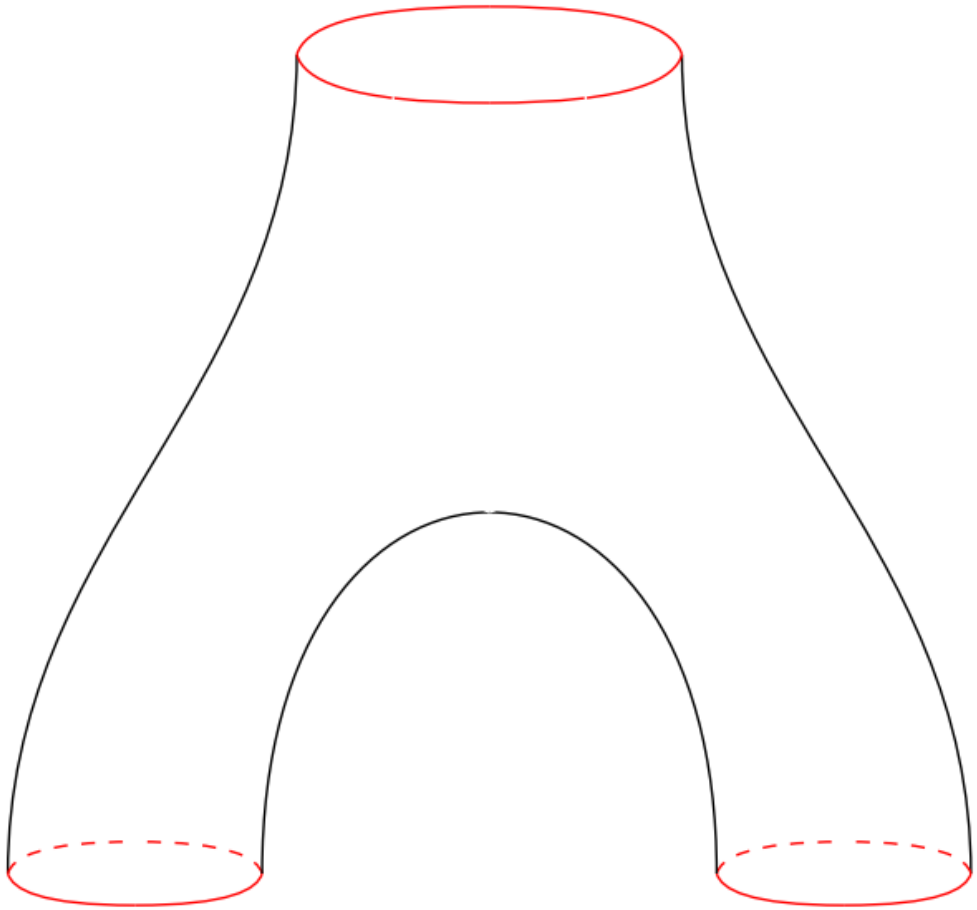
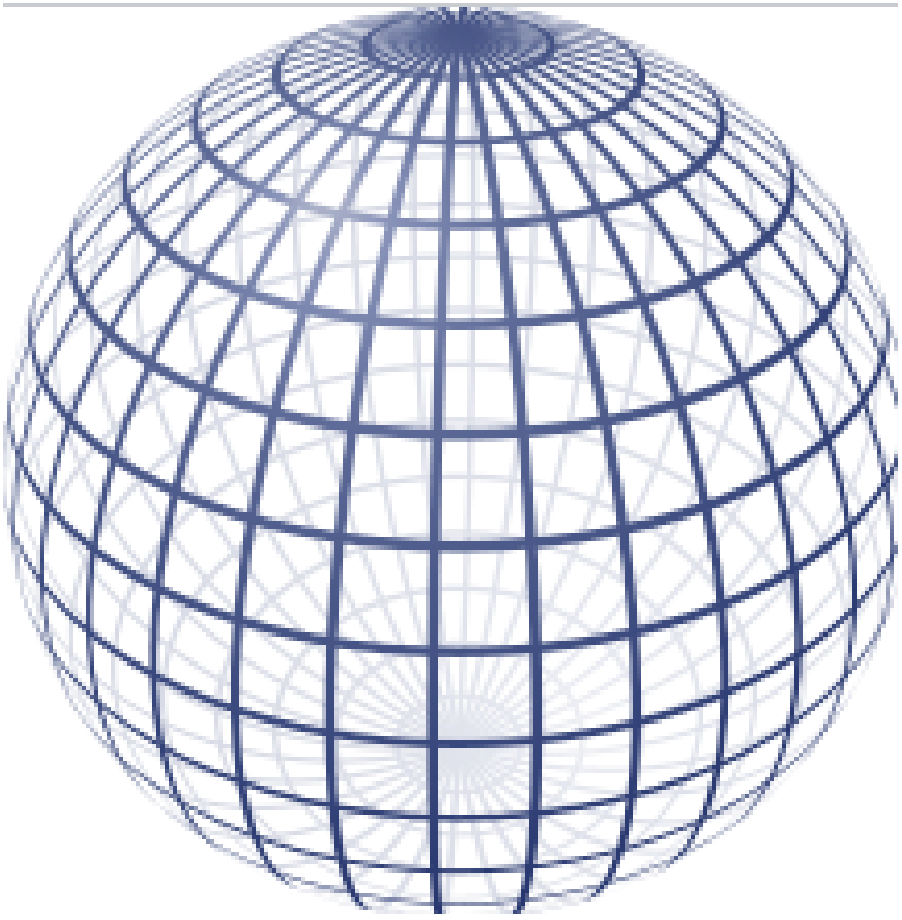


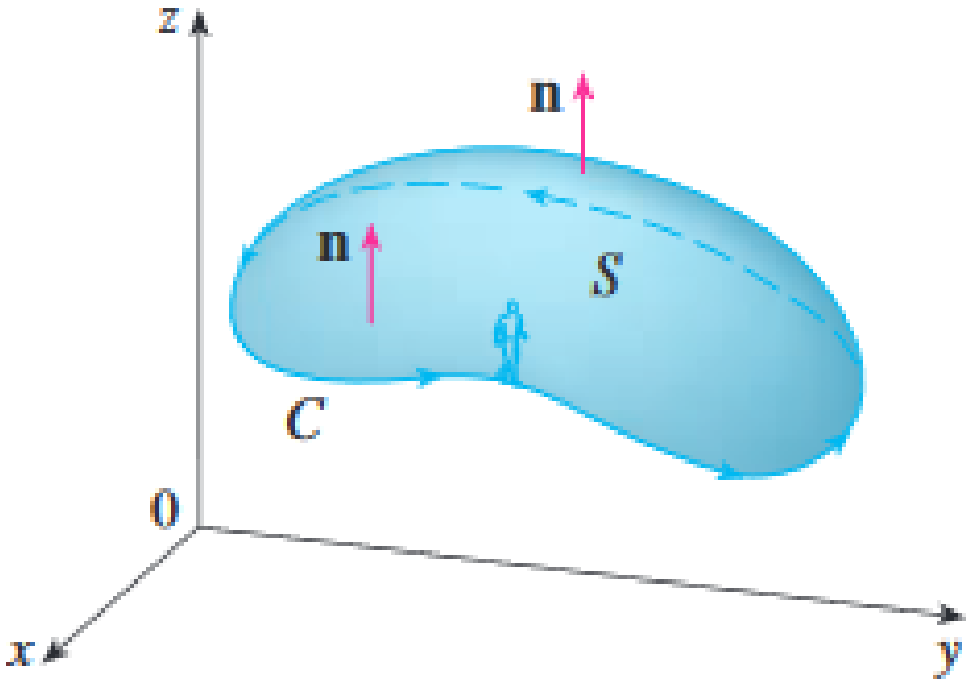
http://tutorial.math.lamar.edu/Classes/CalcIII/StokesTheorem.aspx

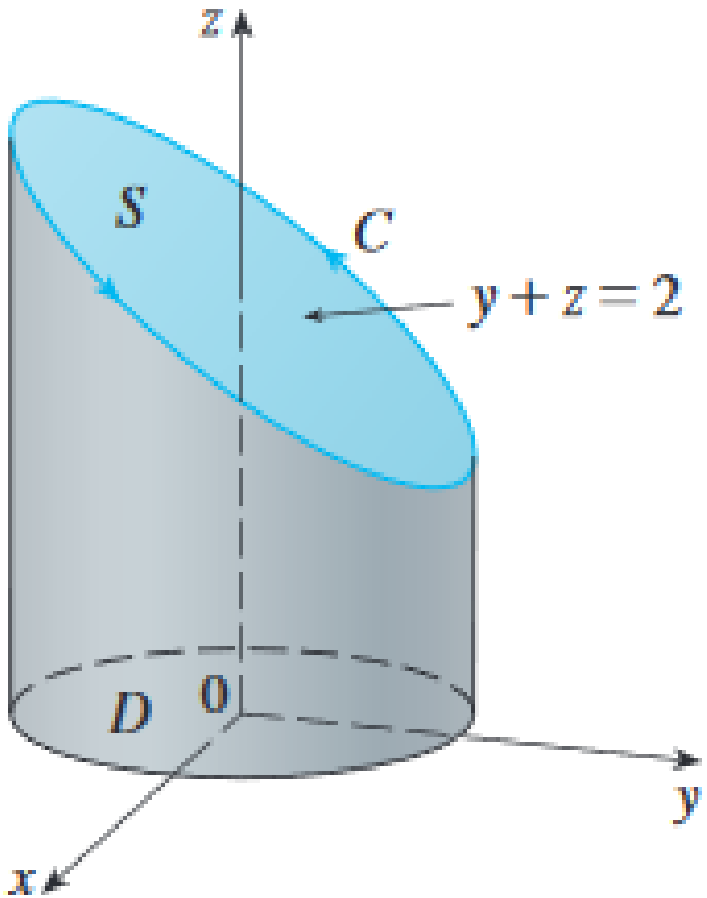


https://en.wikipedia.org/wiki/Pair_of_pants_(mathematics)



https://simple.wikipedia.org/wiki/Sphere




z
S
C
y + z = 2
D
0
y
x

Stokes' Theorem Practice Problems

1. Use line integrals to find $\iint_{\mathcal{S}} \text{curl}(\mathbf{F}) \cdot d\mathbf{S}$ where $\mathbf{F} = \langle yz, xz, xy \rangle$ and $\mathcal{S}$ is the cylinder $x^2 + y^2 = 1$ with $1 \leq z \leq 4$ with outward-pointing normal vectors.

2. Use Stokes' Theorem to find $\oint_{\mathcal{C}} \langle yz, xy, xz \rangle \cdot d\mathbf{r}$ where $\mathcal{C}$ is the square with vertices $(0, 0, 2)$, $(1, 0, 2)$, $(1, 1, 2)$, and $(0, 1, 2)$ oriented counterclockwise.

Challenge Problems

1. Use line integrals to find $\iint_{\mathcal{S}} \text{curl}(\mathbf{F}) \cdot d\mathbf{S}$ where $\mathbf{F} = \langle yz, -xz, z^3 \rangle$ and $\mathcal{S}$ is the cone $z = \sqrt{x^2 + y^2}$ with $1 \leq z \leq 3$ with upward-pointing normal vectors.
2. Let $\mathbf{F} = \langle y, -x, zx^3y^2 \rangle$. Evaluate $\iint_{\mathcal{S}} (\nabla \times \mathbf{F}) \cdot \mathbf{n} dA$ where $\mathcal{S}$ is the surface defined by $x^2 + y^2 + z^2 = 1$, $z \leq 0$ oriented with normal pointing outward.