Vector Line Integrals

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May 2, 2018

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Vector Line Integral Practice Problems

1. Compute $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle 4, y \rangle$ and $\mathcal{C}$ is the quarter circle $x^2 + y^2 = 1$ with $x \leq 0$, $y \leq 0$ oriented counterclockwise.

2. Compute $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \left\langle e^{y-x}, e^{2x} \right\rangle$ and $\mathcal{C}$ is the piecewise path from $(1,1)$ to $(2,2)$ to $(0,2)$.

Challenge Problems

1. Compute $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \left\langle \frac{-y}{(x^2+y^2)^2}, \frac{x}{(x^2+y^2)^2} \right\rangle$ and $\mathcal{C}$ is the circle with radius R centered at the origin and oriented counterclockwise.
2. Let $\mathcal{C}$ be a curve and $\mathbf{T}$ be the unit tangent vector. What is $\int_{\mathcal{C}} \mathbf{T} \cdot d\mathbf{r}$?
3. Let $\mathcal{C}_1$ and $\mathcal{C}_2$ be two paths with the same endpoints and $\mathcal{C}$ be the curve that first moves along $\mathcal{C}_1$ and then moves along $\mathcal{C}_2$ in the opposite direction. Show that for any vector field $\mathbf{F}$, if $\int_{\mathcal{C}_1} \mathbf{F} \cdot d\mathbf{r} = \int_{\mathcal{C}_2} \mathbf{F} \cdot d\mathbf{r}$, then $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r} = 0$.

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