Green's Theorem

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Green's Theorem Practice Problems

1. Let $\mathcal{C}$ be the rectangle with vertices $(1, 1)$, $(3, 1)$, $(1, 4)$, and $(3, 4)$. Evaluate $\oint_{\mathcal{C}} (\ln(x) + y)dx - x^2 dy$.

2. Find $\oint_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle x + y, x^2 - y \rangle$ and $\mathcal{C}$ is the boundary of the region enclosed by $y = x^2$ and $y = \sqrt{x}$ for $0 \leq x \leq 1$.

Challenge Problems

1. Use line integrals to find the area of the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

2. Suppose that f is a function such that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$ over the region $\mathcal{D}$. Prove $\int_{\partial \mathcal{D}} \frac{\partial f}{\partial y} dx - \frac{\partial f}{\partial x} dy = 0$.

3. Let $\text{curl}_z(\mathbf{F})$ be the z component of the curl. Show $\oint_{\partial \mathcal{D}} \mathbf{F} \cdot d\mathbf{r} = \iint_{\mathcal{D}} \text{curl}_z(\mathbf{F}) dA$.