Change of Variables Day 1

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Linear Maps Practice Problems

   1 Let $G(u, v) = (2u + v, 5u + 3v)$. Write the image of the line through $(u, v) = (1, 1)$ and $(u, v) = (1, -1)$ in slope-intercept form.

   2 Let $G(u, v) = (2u + v, 5u + 3v)$. Show that G maps the line $v = mu$ to the line $y = \frac{5+3m}{2+m}x$.

Challenge Problems

   1 Let $\mathcal{D}$ be the square $[0, 1] \times [0, 1]$. Find a linear map G such that $G(\mathcal{D})$ is the parallelogram with vertices $(0, 0)$, $(-1, 3)$, $(-2, 0)$, and $(-1, -3)$.

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Jacobian Practice Problems

   ❸ Find the Jacobian for $G(u, v) = (2u + v, 5u + 3v)$.

   ❹ Find the Jacobian for $G(r, s) = (rs, r + s)$.

Challenge Problems

   ❷ Let $G(u, v) = (Au + Bv, Cu + Dv)$ be a linear mapping. Show that for $\mathcal{R} = [0, 1] \times [0, 1]$, $\text{Area}(G(\mathcal{R})) = |\text{Jac}(G)|\text{Area}(\mathcal{R})$.

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