Double Integrals over General Regions

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Math13

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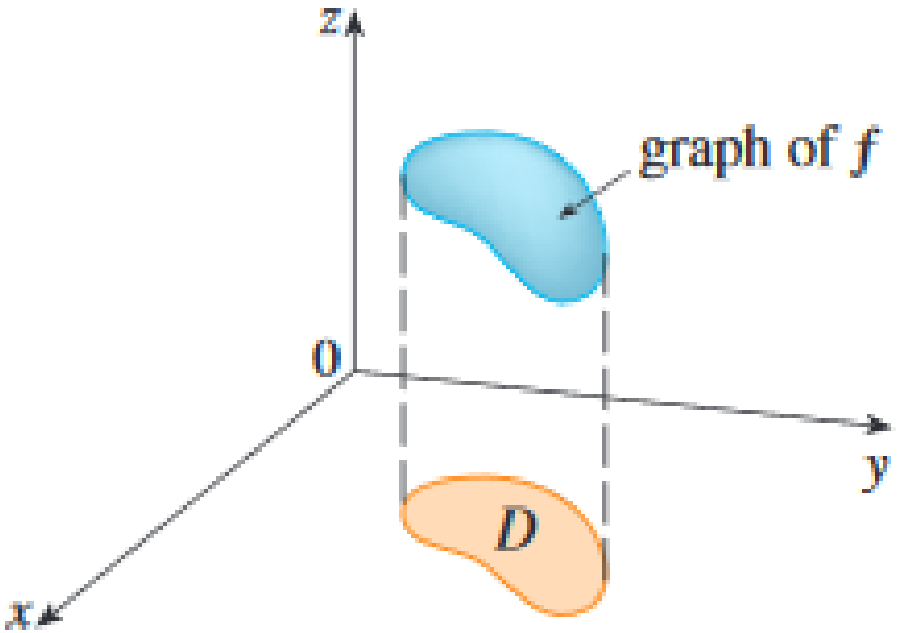
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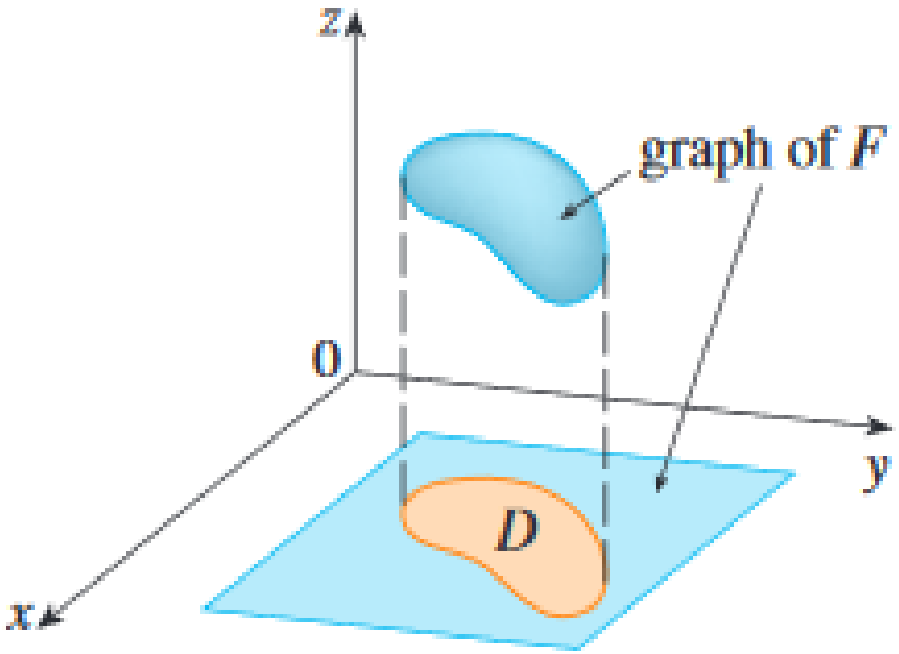
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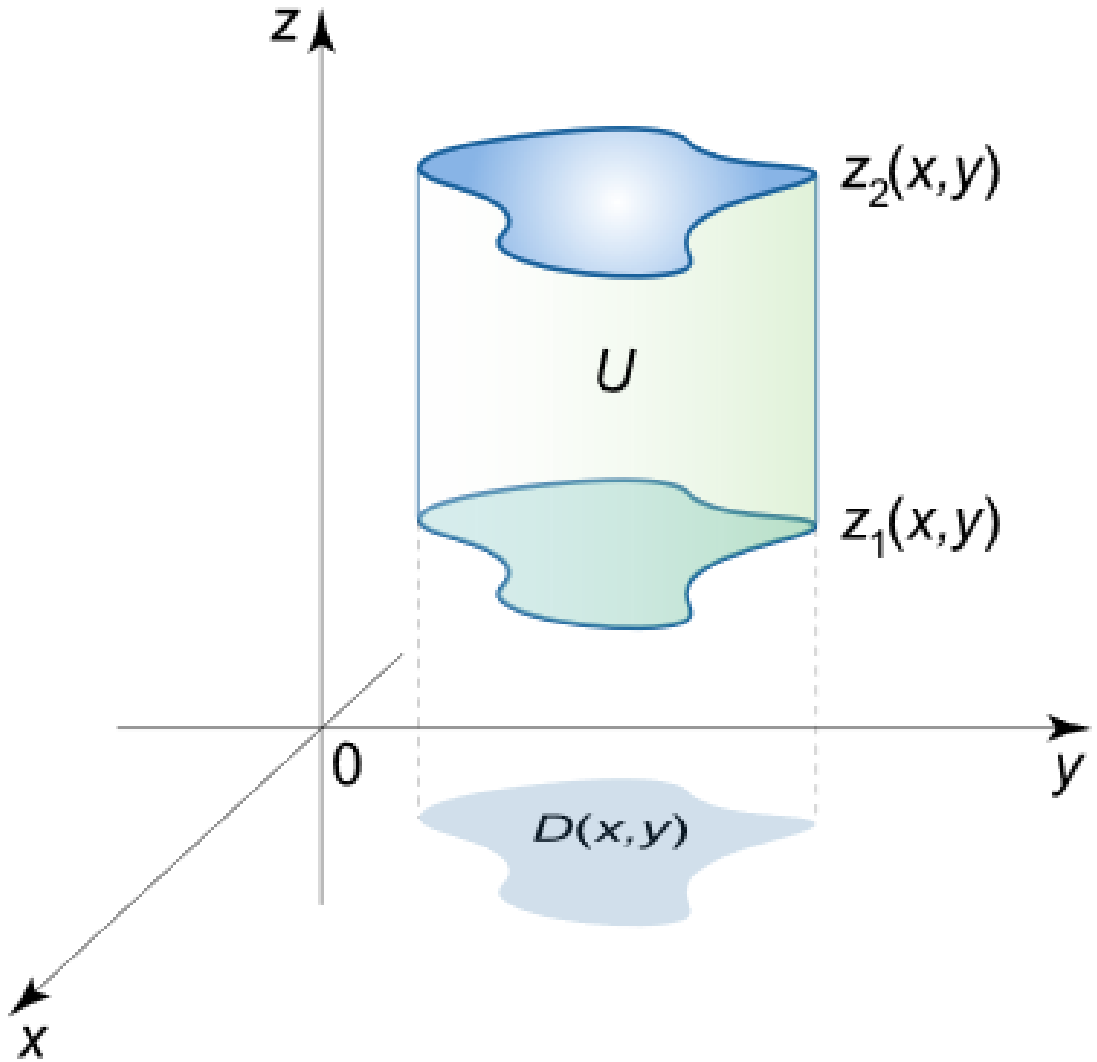

Practice Problems

1. Integrate $f(x, y) = x$ over the region bounded by $y = x^2$ and $y = x + 2$.

2. Sketch the domain of integration for $\int_0^4 \int_x^4 f(x, y) dy dx$, and then express as an iterated integral in the opposite order.

3. Find the volume of the region bounded by $z = 50 - 10y$, $z = 10$, $y = 0$, and $y = 4 - x^2$.

Challenge Problems

1. Let $\mathcal{D}$ be the domain bounded by $y = x^2 + 1$ and $y = 2$. Prove the inequality $\frac{4}{3} \leq \iint_{\mathcal{D}} (x^2 + y^2) dA \leq \frac{20}{3}$.
2. Verify the Mean Value Theorem for $f(x, y) = e^{x-y}$ on the triangle bounded by $y = 0$, $x = 1$, and $y = x$.
3. Is it true that $\iint_{\mathcal{D}} f(x)g(y) dy dx = \left(\int_a^b f(x) dx \right) \left(\int_{h_1(a)}^{h_2(b)} g(y) dy \right)$ for vertically simple regions? Why or why not?
4. Use integrals to calculate the volume of a cone of base radius r and height h.