Iterated Integrals

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Math13

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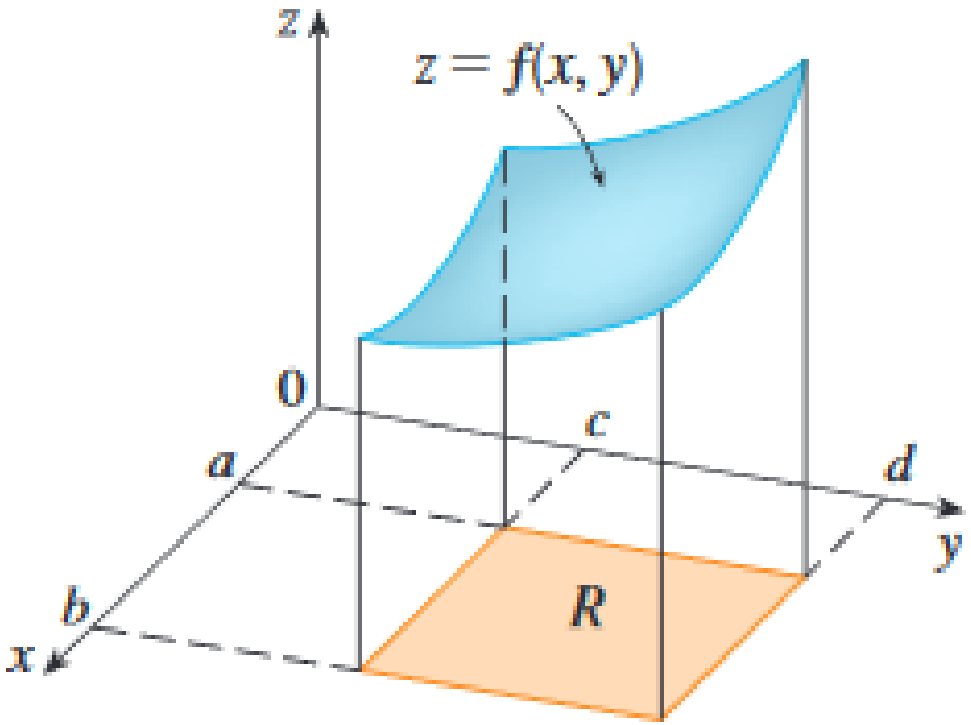
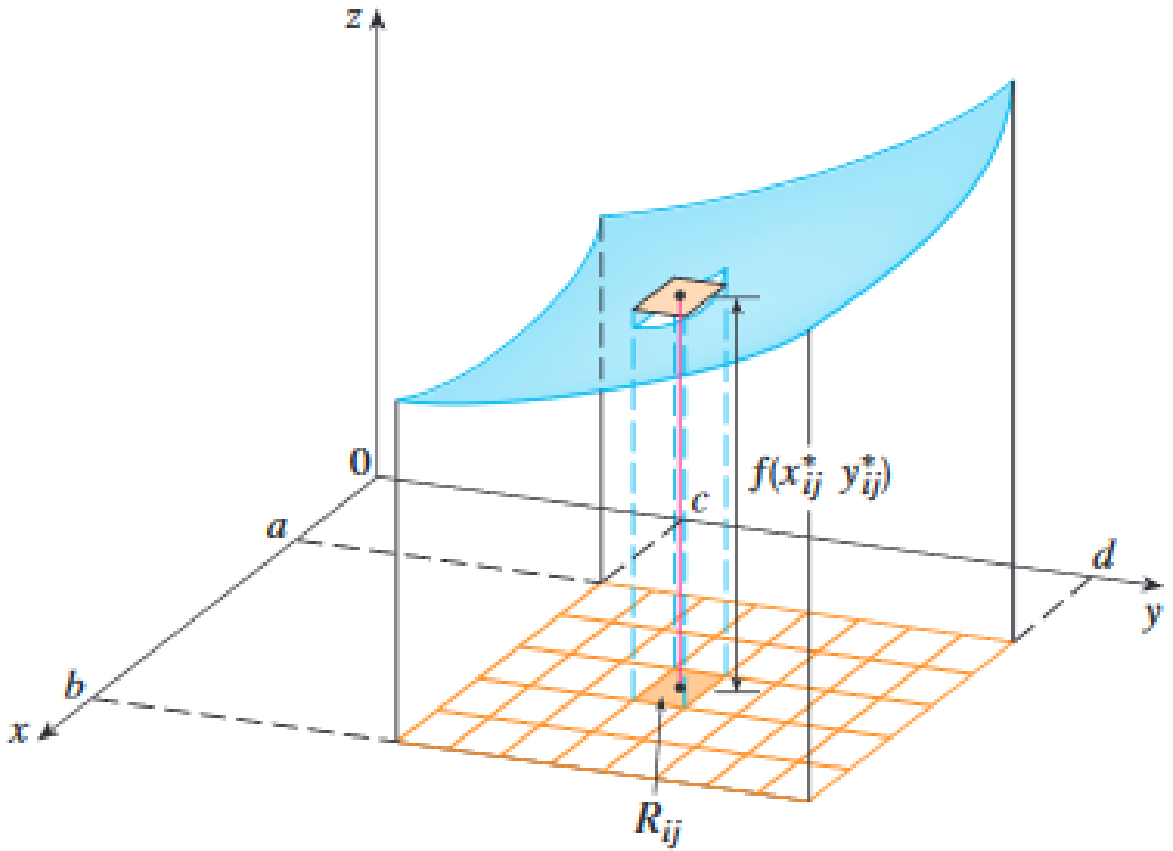
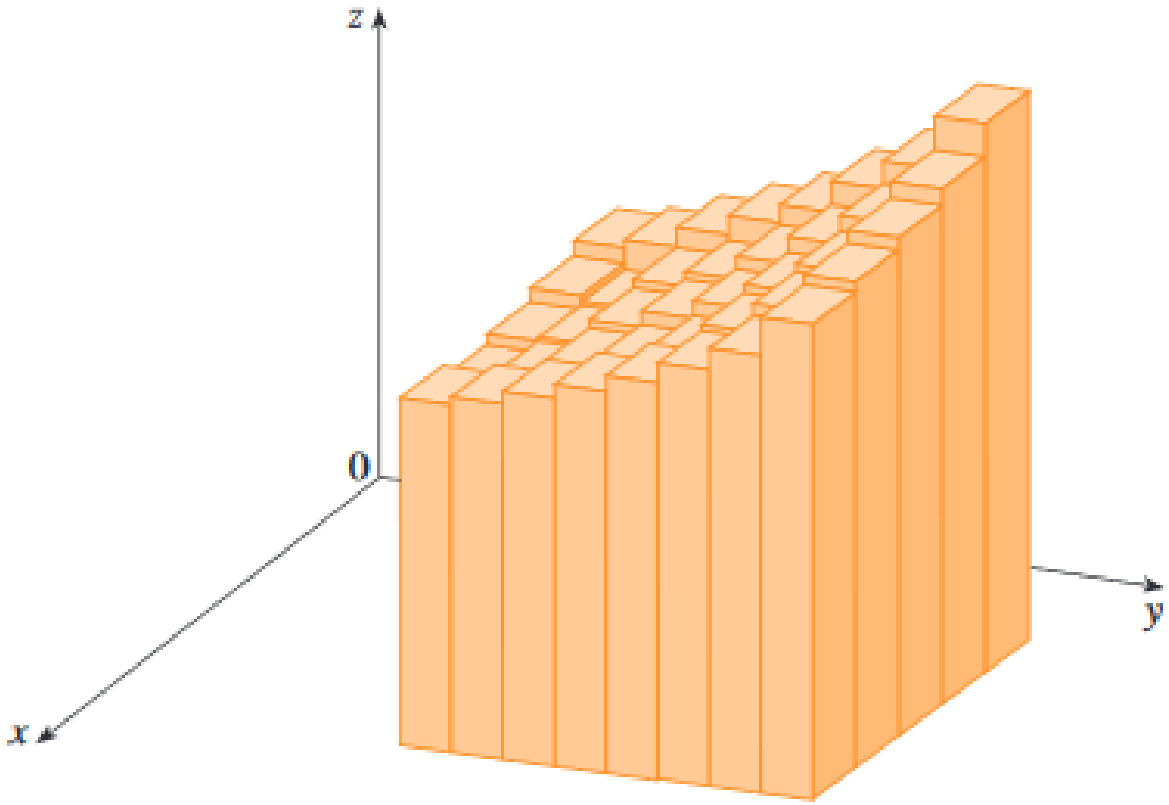
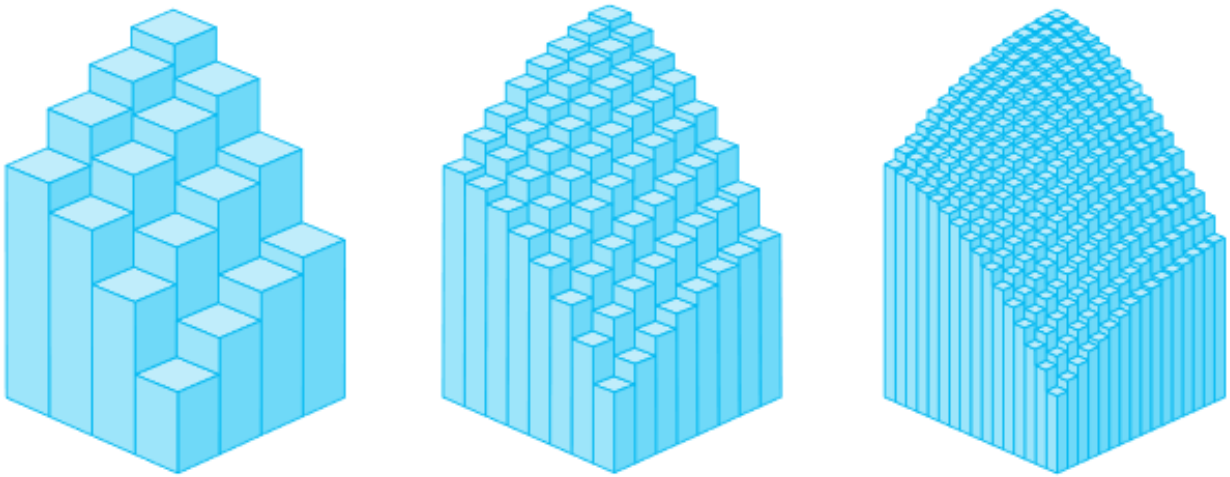

z
z = f(x, y)
0
c
a
d
y
b
R
x

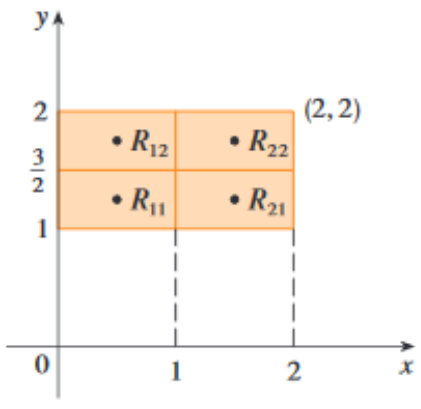

z
0
a
b
x
c
d
y
$f(x_{ij}^*, y_{ij}^*)$
R_{ij}

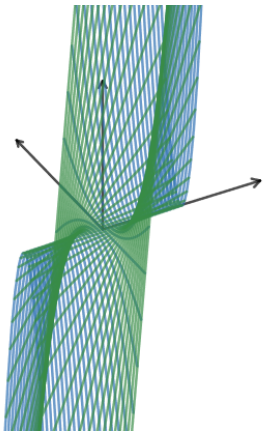

z
0
x
y



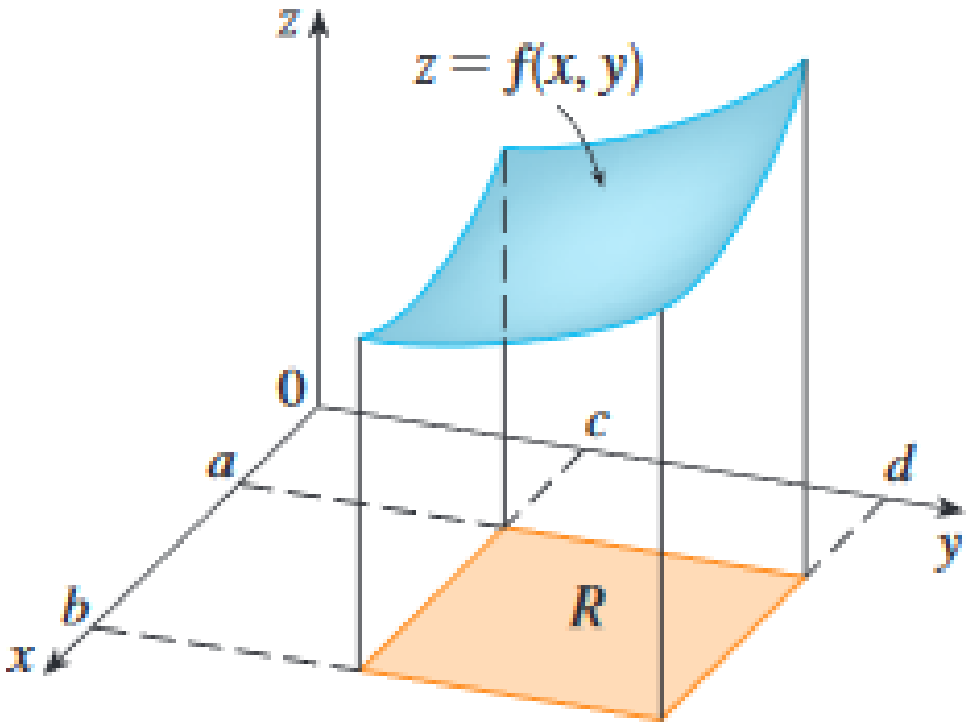
Riemann Sum Practice

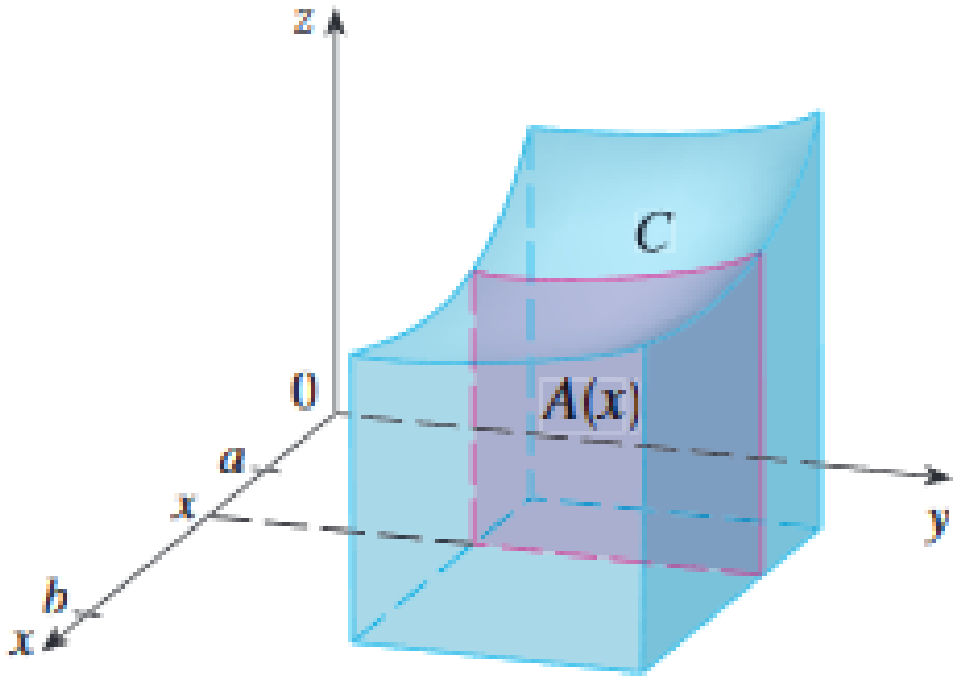
1. Approximate $\int\int_R f(x,y)dA$ where R is the rectangle $[0,2]\times[1,2]$ in the figure below, and $f(x,y)=x-2y$. The dots in the figure tell us what x and y to plug into $f(x,y)$ to get a representative value for each of the four pieces that we've partitioned our rectangle into.

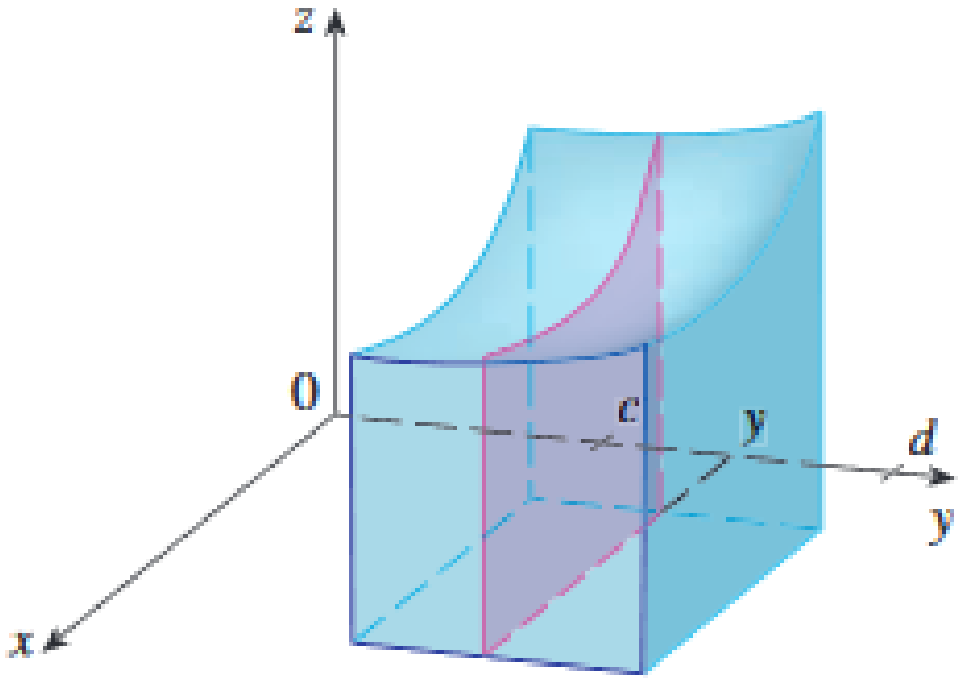




2. Without calculating, explain why $\int\int_R xy^2dA=0$ where $R=[-1,1]\times[-1,1]$ (see figure above).


z
z = f(x, y)
0
c
a
d
y
b
R
x


z
C
0
A(x)
a
x
y
b
x



Iterated Integral Practice

3. Evaluate the double integrals:
   (a) $\int_0^1 \int_0^2 (x + 4y^3) dx dy$

   (b) $\int_0^2 \int_0^1 (x + 4y^3) dy dx$

4. Evaluate $\int_{-1}^1 \int_0^\pi x^2 \sin(y) dy dx$.

5. Let $g(x)$ be a function with x as the only variable, and $h(y)$ be a function with y as the only variable. Explain why

$$\int_a^b \int_c^d g(x)h(y) dy dx = \int_a^b g(x) dx \int_c^d h(y) dy.$$