Math 13 Spring 2017, Midterm Exam II

Tuesday, May 9th

Name: ______________________________

Instructor: ☐ Prof. Clare ☐ Prof. Song

INSTRUCTIONS

This is a closed book, closed notes exam.

There are 9 problems. You have 2 hours.

Use of calculators is not permitted.

On each question you must show your work. No credit is given for solutions without supporting calculations.

The Honor Principle requires that you neither give nor receive any aid on this exam.

GOOD LUCK!

FERPA waiver: By my signature I relinquish my FERPA rights in the following context: This exam paper may be returned en masse with others in the class and I acknowledge that I understand my score may be visible to others. If I choose not to relinquish my FERPA rights, I understand that I will have to present my student ID at my instructors office to retrieve my examination paper.

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1	2	3	4	5	6	7	8	9	TOTAL

(1) (*9 points*) For each of the following assertions, select the correct ending.

(a) Let $\mathbb{D}$ be the disk with radius 1 and center at the origin in the uv-plane and G the mapping defined by

$$G(u, v) = (u + 2v, -u + 4v).$$

The area of $G(\mathbb{D})$ is...

☐ 6π.

☐ $\frac{\pi}{6}$.

☐ π.

☐ 2π.

☐ $\frac{\pi}{2}$.

☐ none of the above.

(b) Let $f(x, y, z) = y^3$. Then $\text{grad}\,(\text{div}\,(\text{grad}\, f)) = \dots$

☐ $6\mathbf{j}$.

☐ 6.

☐ $6y$.

☐ $\langle 0, 3y^2, 0 \rangle$.

☐ none of the above.

(c) Let $\mathbf{F}$ be a vector field. Then $\text{grad}\,(\text{curl}\,\mathbf{F})$ is...

☐ a potential for $\mathbf{F}$.

☐ the divergence of $\mathbf{F}$.

☐ a vector field.

☐ not well-defined.

☐ none of the above.

(2) (*8 points*) Determine the image of the triangular region with vertices $(0,0)$, $(1,1)$ and $(0,1)$ under the transformation $G(u,v) = (u^2, v)$.

Your answer must contain two figures: the original region and its image, each in the appropriate coordinate plane with all the pertinent information.

(3) (*10 points*) The mapping $G(u,v) = \left(\frac{1}{2}(u+v), \frac{1}{2}(u-v)\right)$ transforms the trapezoidal region $\mathcal{S}$ with vertices $(-2,2)$, $(2,2)$, $(1,1)$ and $(-1,1)$ in the uv-plane into the trapezoidal region $\mathcal{R}$ with vertices $(1,0)$, $(2,0)$, $(0,-2)$ and $(0,-1)$ in the xy-plane.

Use this change of variables to evaluate the integral $\iint_{\mathcal{R}} e^{\frac{x+y}{x-y}}\, dA$.

(4) (*6 points*) The volume of the solid ellipsoid $\mathcal{E}: \quad \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} \leq 1$ is given by:

$$\text{Vol}(\mathcal{E}) = \iiint_{\mathcal{E}} 1 \, dV.$$

Use one of the following changes of variables and the fact that the ball with center at the origin and radius 1 has volume $\dfrac{4}{3}\pi$ to determine $\text{Vol}(\mathcal{E})$.

$$G_1(u, v, w) = \left(\frac{u}{a}, \frac{v}{b}, \frac{w}{c}\right)$$
$$G_2(u, v, w) = (au, bv, cw)$$
$$G_3(u, v, w) = (\sqrt{u}, \sqrt{v}, \sqrt{w})$$

(5) (*8 points*) Evaluate the integral

$$\int_{\mathcal{C}} 18y^3\, ds$$

where $\mathcal{C}$ is the curve in the plane parameterized by $x(t) = t^3$, $y(t) = t$ with $0 \le t \le 1$.

Hint: You may use without proof any of the following facts:

$$\int_0^1 \sqrt{u}\, du = \frac{2}{3} \quad , \quad \int_0^9 \sqrt{u}\, du = 18 \quad , \quad \int_1^{10} \sqrt{u}\, du = \frac{2(10\sqrt{10} - 1)}{3} \quad , \quad \int_1^9 \sqrt{u}\, du = \frac{52}{3}$$

(6) (*8 points*) Evaluate the integral

$$\int_{\mathcal{L}} xe^{yz}\, ds$$

where $\mathcal{L}$ is the line segment from $(0, 0, 0)$ to $(1, 2, 3)$.

(7) (*7 points*) Evaluate the integral of the vector field

$$\mathbf{F}(x, y, z) = \left(x + y + \frac{z}{4}\right)\mathbf{i} + (y - x^3)\mathbf{j} + \ln\left(\frac{x+z}{y+1}\right)\mathbf{k}$$

along the curve $\mathbf{r}(t) = t\mathbf{i} + t^3\mathbf{j} + 4\mathbf{k}$ for $0 \le t \le 1$.

(8) (*7 points*) The vector field $\mathbf{F}(x, y) = (3 + 2xy)\mathbf{i} + (x^2 - 3y^2)\mathbf{j}$ is conservative (**do not prove it**). Find a potential for $\mathbf{F}$.

(9) (*7 points*) Let $f(x, y) = xe^y$ and $\mathbf{F} = \nabla f$. Evaluate $\displaystyle\int_\Gamma \mathbf{F} \cdot d\mathbf{r}$, where Γ is given by

$$\mathbf{r}(t) = te^t\mathbf{i} + \sqrt{1 + 3t}\,\mathbf{j}$$

for $0 \leq t \leq 1$.

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