Homework 8

Due date: May 29, 2017

Problem 1: Use Stokes' Theorem to calculate the integral $\iint_\Sigma \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$ where

$$\mathbf{F}(x, y, z) = \langle xz\,,\, yz\,,\, xy \rangle$$

and Σ is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the xy-plane.
Use the orientation on Σ given by downward-pointing normal vectors.

Problem 2: Evaluate the circulation of the vector field

$$\mathbf{F}(x, y, z) = yz \cdot \mathbf{i} + 2xz \cdot \mathbf{j} + e^{xy} \cdot \mathbf{k}$$

along the circle Γ with equation $x^2+y^2 = 16$ in the $z = 5$ plane, oriented counterclockwise as seen from above.

Problem 3: Let

$$\mathbf{F} = (2x + y \cos \sqrt{z}) \cdot \mathbf{i} + (y - \sin xz) \cdot \mathbf{j} + (x + y - z) \cdot \mathbf{k}.$$

Let Σ be the surface consisting of the paraboloid $y = x^2 + z^2, 0 \leq y \leq 1$ and the disk $x^2 + z^2 \leq 1, y = 1$. Find

$$\iint_\Sigma \mathbf{F} \cdot d\mathbf{S}.$$

Use normal vectors pointing in the negative y-direction on the paraboloid and in the positive y-direction on the disk.